Calculus

Name ______________________

Winter Break Assignment

Evaluate each limit.

1) $\lim_{x \to 2} -\frac{x-2}{x^2 - 5x + 6}$

1

2) $\lim_{x \to 0} \frac{\sin(4x)}{\sin(5x)}$

$\frac{4}{5}$

3) $\lim_{x \to 0} \frac{1 - \cos(2x)}{5x}$

0

4) $\lim_{x \to \infty} \frac{\sqrt{2x^2 + 3}}{2x}$

$\frac{\sqrt{2}}{2}$

5) $\lim_{x \to \infty} (x^3 - 2x^2 - 3)$

∞

6) $\lim_{x \to -\infty} -e^{\frac{1}{x}}$

-1

Use logarithmic differentiation to differentiate each function with respect to *x*.

7) $y = 3x^{x^5}$

$$\frac{dy}{dx} = y(5x^4 \ln x + x^4)$$
$$= 3x^{x^5 + 4}(5 \ln x + 1)$$

8) $y = 2x^{x^4}$

$$\frac{dy}{dx} = y(4x^3 \ln x + x^3)$$
$$= 2x^{x^4 + 3}(4 \ln x + 1)$$

9) $y = 5x^{2x}$

$$\frac{dy}{dx} = y(2 \ln x + 2)$$
$$= 10x^{2x}(\ln x + 1)$$

10) $y = x^{3x}$

$$\frac{dy}{dx} = y(3 \ln x + 3)$$
$$= 3x^{3x}(\ln x + 1)$$

11) $y = 2x^{x^5}$

$$\frac{dy}{dx} = y(5x^4 \ln x + x^4)$$
$$= 2x^{x^5 + 4}(5 \ln x + 1)$$

Differentiate each function with respect to *x*.

12) $y = \ln \left(\frac{5x^5}{x^2 + 3}\right)^5$

$$\frac{dy}{dx} = 5\left(\frac{1}{5x^5} \cdot 25x^4 - \frac{1}{x^2 + 3} \cdot 2x\right)$$
$$= \frac{15(x^2 + 5)}{x(x^2 + 3)}$$

13) $y = \ln \left(\frac{4x^5}{x^3 + 3}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{1}{4x^5} \cdot 20x^4 - \frac{1}{x^3 + 3} \cdot 3x^2\right)$$
$$= \frac{3(2x^3 + 15)}{x(x^3 + 3)}$$

14) $y = \ln \left(\frac{5x^2}{5x^3 - 4}\right)^4$

$$\frac{dy}{dx} = 4\left(\frac{1}{5x^2} \cdot 10x - \frac{1}{5x^3 - 4} \cdot 15x^2\right)$$
$$= \frac{4(-5x^3 - 8)}{x(5x^3 - 4)}$$

15) $y = \ln \left(\frac{5x^2}{x^3 + 2}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{1}{5x^2} \cdot 10x - \frac{1}{x^3 + 2} \cdot 3x^2\right)$$
$$= \frac{3(-x^3 + 4)}{x(x^3 + 2)}$$

16) $y = \ln \left(\frac{x^4}{2x^3 - 3}\right)^5$

$$\frac{dy}{dx} = 5\left(\frac{1}{x^4} \cdot 4x^3 - \frac{1}{2x^3 - 3} \cdot 6x^2\right)$$
$$= \frac{10(x^3 - 6)}{x(2x^3 - 3)}$$

17) $y = \log_4 \left(\frac{5x^5}{x^4 - 3}\right)^4$

$$\frac{dy}{dx} = 4\left(\frac{1}{5x^5 \ln 4} \cdot 25x^4 - \frac{1}{(x^4 - 3)\ln 4} \cdot 4x^3\right)$$
$$= \frac{4(x^4 - 15)}{x(x^4 - 3)\ln 4}$$

18) $y = \log_2 \left(\frac{4x^3}{x^4 + 5} \right)^3$

$$\frac{dy}{dx} = 3\left(\frac{1}{4x^3 \ln 2} \cdot 12x^2 - \frac{1}{(x^4 + 5)\ln 2} \cdot 4x^3 \right)$$
$$= \frac{3(-x^4 + 15)}{x(x^4 + 5)\ln 2}$$

19) $y = \log_4 \left(\frac{x^3}{3x^4 - 2} \right)^4$

$$\frac{dy}{dx} = 4\left(\frac{1}{x^3 \ln 4} \cdot 3x^2 - \frac{1}{(3x^4 - 2)\ln 4} \cdot 12x^3 \right)$$
$$= \frac{12(-x^4 - 2)}{x(3x^4 - 2)\ln 4}$$

20) $y = \log_2 \left(\frac{2x^3}{x^2 - 2} \right)^4$

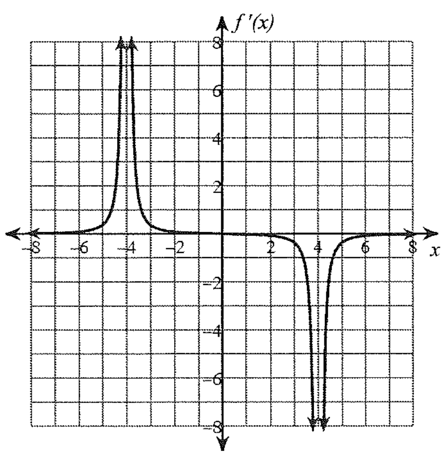
$$\frac{dy}{dx} = 4\left(\frac{1}{2x^3 \ln 2} \cdot 6x^2 - \frac{1}{(x^2 - 2)\ln 2} \cdot 2x \right)$$
$$= \frac{4(x^2 - 6)}{x(x^2 - 2)\ln 2}$$

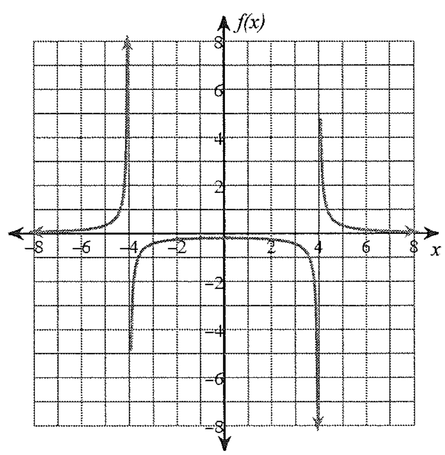
21) $y = \log_3 \left(\frac{x^2}{4x^3 - 1} \right)^2$

$$\frac{dy}{dx} = 2\left(\frac{1}{x^2 \ln 3} \cdot 2x - \frac{1}{(4x^3 - 1)\ln 3} \cdot 12x^2 \right)$$
$$= \frac{4(-2x^3 - 1)}{x(4x^3 - 1)\ln 3}$$

Given the graph of $f'(x)$, sketch a possible graph of $f(x)$.

22)





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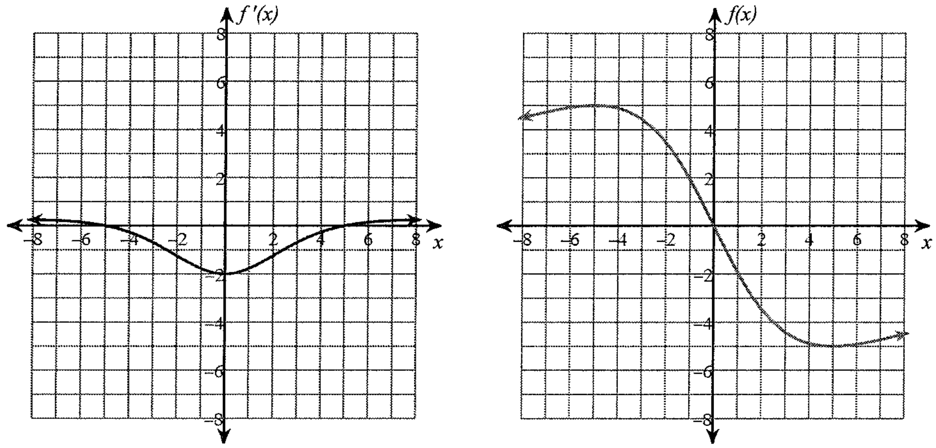

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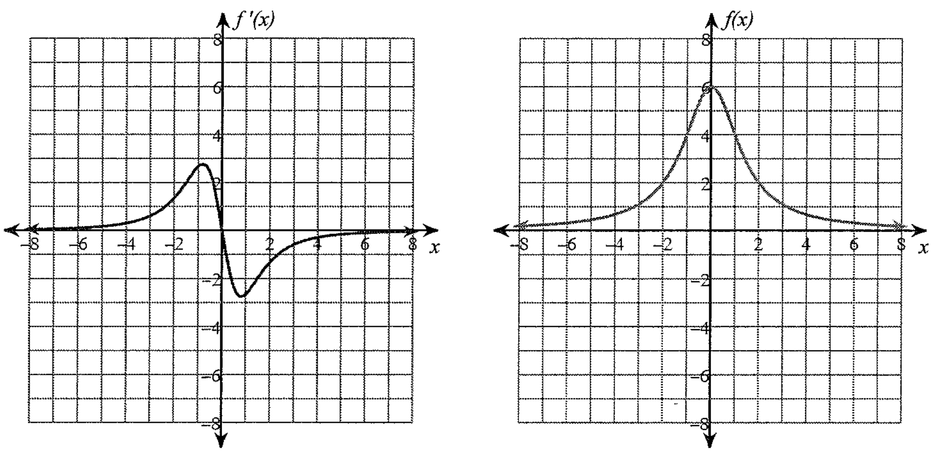

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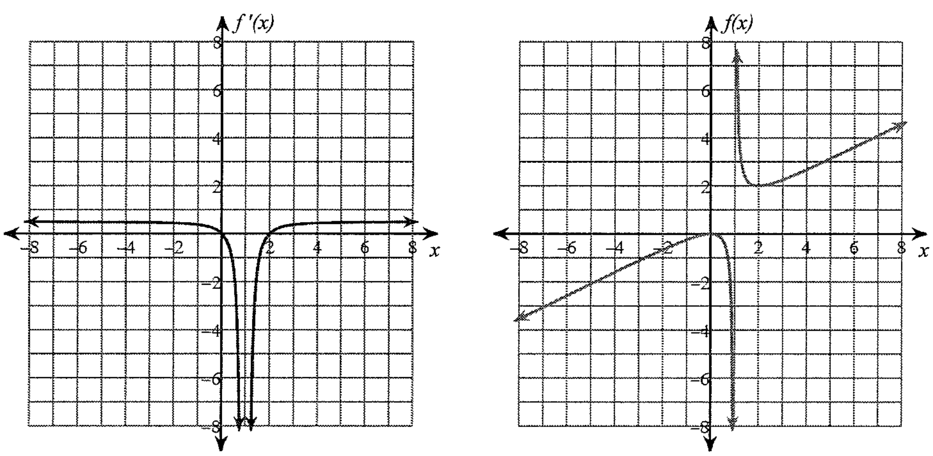


Solve each optimization problem.

26) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(5, 0)$?

$\left(\frac{9}{2}, \frac{3\sqrt{2}}{2}\right)$

Winter Break Assignment Answers

Topic 1: 1. exponential functions (e^x) grow faster than power functions (x^2)

2. e^x

Topic 2:

3. $$\lim_{x \to \infty} \frac{4x^7 + 2e^{0.2x}}{2x^{10} + e^{0.2x}} = 2$$

(exponential functions grow faster)

4. $$\lim_{x \to -\infty} \frac{4x^7 + 2e^{0.2x}}{2x^{10} + e^{0.2x}} = 0$$ (bottom heavy, also it's as $x \to -\infty$)

5. $$\lim_{x \to \infty} \frac{\text{Log}[x]}{x^2} = 0$$

6. $$\lim_{x \to -\infty} \frac{\text{log}[x]}{x^2} = \infty$$

power functions grow faster than log functions. Try graphing in mathable (not calculator) with domain going from $(0, 10^{200})$

7. $$\lim_{x \to \infty} \frac{e^x}{4x^2} = \infty$$

↓ exponential functions grow faster than power functions

8. $$\lim_{x \to -\infty} \frac{e^x}{4x^2} = 0$$

9. $$\lim_{x \to \infty} \frac{4x^2}{e^x} = 0$$

10. $$\lim_{x \to -\infty} \frac{4x^2}{e^x} = \infty$$

Topic 3:

1. A

2. B

Worksheet 84

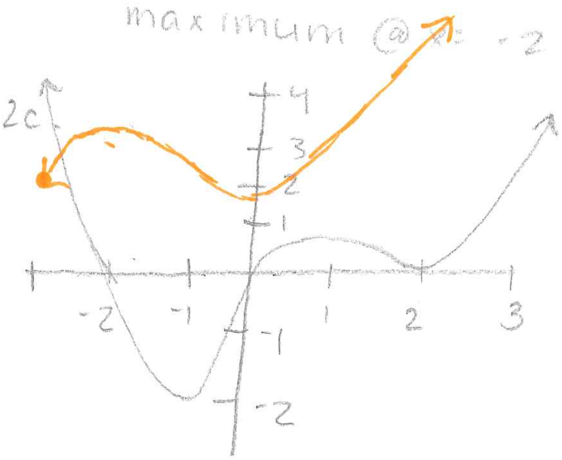
1a. increasing $(-\infty, -3.3)$, $(-1.5, .7)$ (about) $f'(x) > 0$
decreasing $(-3.3, -1.5)$ $(.7, \infty)$ $f'(x) < 0$

1b. minimums @ $x = -1.5$ b/c $f'(x)$ changes from − to +
maximums @ $x = -3.3$ and $x = .7$ b/c $f'(x)$ changes from + to −.

2.a. increasing $(-\infty, -2)$ $(0, \infty)$ $f'(x) > 0$
decreasing $(-2, 0)$ $f'(x) < 0$

2b. minimum @ $x = 0$ b/c $f'(x)$ changes from − to +
maximum @ $x = -2$ b/c $f'(x)$ changes from + to −

2c

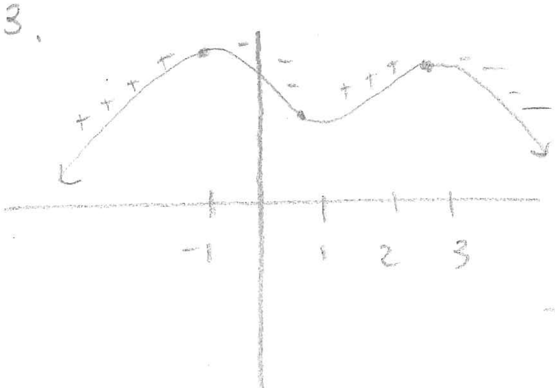


4a. $x = -2$ (max)
$x = 0$ (min)
~~x = 2~~ (doesn't change, so not critical value)

4b. $[-3, -2)$ $(0, 3]$ $f'(x) > 0$

4c. C↑ $(-1, 1)$ $(2, 3]$ $f'(x)$ increasing
C↓ $[-3, -1)$ $(1, 2)$ $f'(x)$ decreasing

3.



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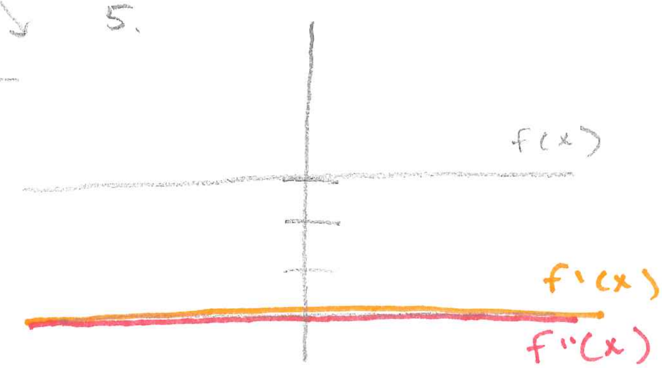

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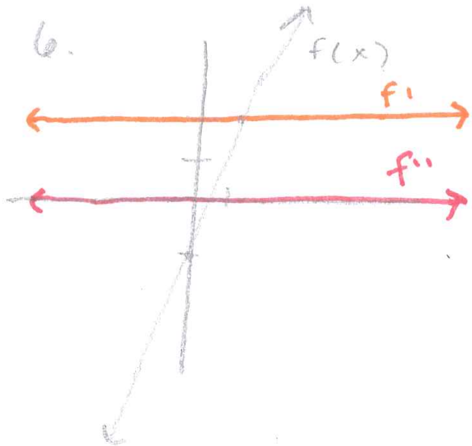

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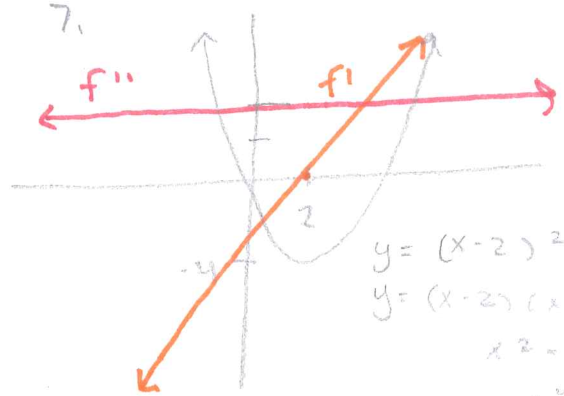


$y = (x-2)^2 - 4$

$y = (x-2)(x-2) - 4$

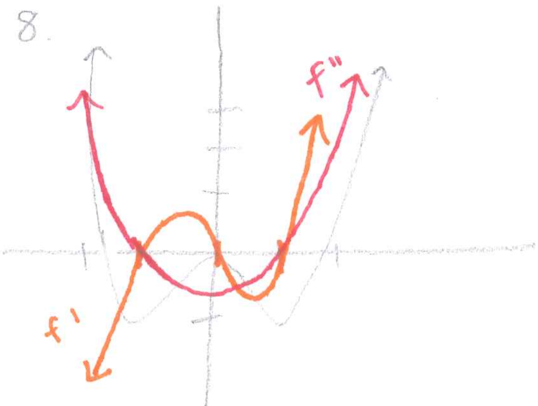
$x^2 - 4x + 4 - 4$

$x^2 - 4x$

$y' = 2x - 4$

$y'' = 2$

8.



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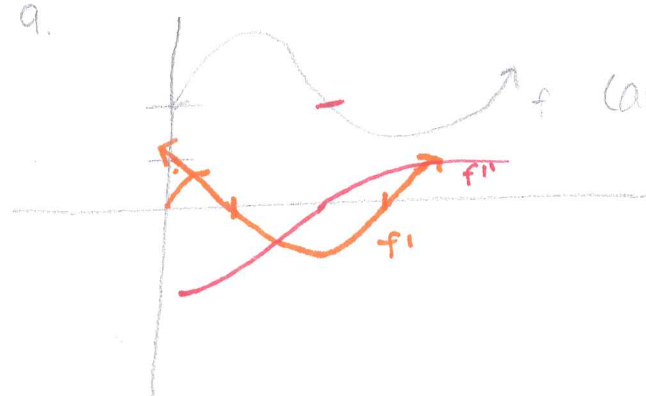
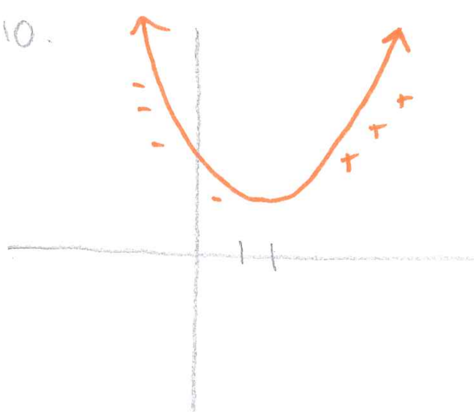


(answers are not too exact here)

10.



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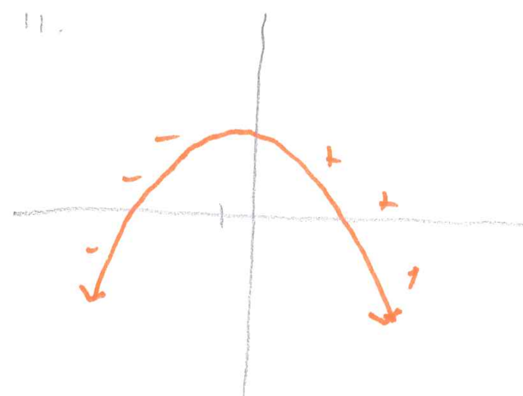
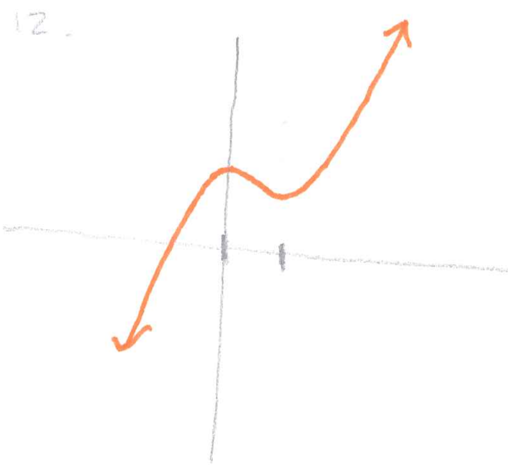


12.



13.