8.1 Sequences and Series

Homework: None

Warm-Up: Find the Sum (88, 90, 96)

88. $\sum_{i=1}^{6}(3i-1)$ = 57

90. $\sum_{k=1}^{5}4$ = 20

92. $\sum_{i=0}^{5}3i^2$

94. $\sum_{j=3}^{5}\frac{1}{j}$

96. $\sum_{k=2}^{5}(k+1)(k-3)$ = 14

98. $\sum_{j=0}^{4}(-2)^j$

Series

Definition of a Series

Consider the infinite sequence $a_1, a_2, a_3, \ldots, a_i, \ldots$.

1. The sum of the first n terms of the sequence is called a **finite series** or the **partial sum** of the sequence and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i.$$

2. The sum of all the terms of the infinite sequence is called an **infinite series** and is denoted by

$$a_1 + a_2 + a_3 + \cdots + a_i + \cdots = \sum_{i=1}^{\infty} a_i.$$

Example 1: Find the 3rd Partial Sum of $\sum_{i=1}^{\infty}\frac{3}{10^i}$

$\frac{3}{10^1} + \frac{3}{10^2} + \frac{3}{10^3}$

.333

Example 2: Find the sum of $\sum_{i=1}^{\infty} \frac{3}{10^i}$ $= \frac{1}{3}$

Arithmetic Sequences

An arithmetic sequence is a list of numbers in which the difference between two consecutive terms is constant. The distance is called d

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The *n*th Term of an Arithmetic Sequence

The *n*th term of an arithmetic sequence has the form

$$a_n = a_1 + (n - 1)d$$

where d is the common difference between consecutive terms of the sequence and a_1 is the first term of the sequence.

Example 1: Find a formula for the *nth* term of the arithmetic sequence whose common difference is 3 and whose first term is 2.

$a_n = 2 + (n-1)3$

$a_n = 2 + 3n - 3$

$a_n = 3n - 1$

Example 2: The fourth term of an arithmetic sequence is $20,$ and the $13th$ term is 65. Write the first several terms of this sequence.

$a_4 = 20$

$a_{13} = 65$

$65 = 20 + 9d$

$45 = 9d$

$d = 5$

$a_3 = 15$

$a_2 = 10$

$a_1 = 5$

Example 3: Find the ninth term of an arithmetic sequence whose first 2 terms are 2 and 9

$d = 7$

$a_9 = 2 + (9-1)7$

$a_9 = 58$

The Sum of a Finite Arithmetic Sequence (See the proof on page 633.)

The sum of a finite arithmetic sequence with n terms is given by

$$S_n = \frac{n}{2}(a_1 + a_n).$$

#terms, 1st, last

Example 4: Find the sum of $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

$S_{10} = \frac{10}{2}(1 + 19) = 100$

Example 4: Find the $150th$ partial sum of the arithmetic sequence $5, 16, 27, 38, 49, \ldots$.

$a_n = a_1 + (n-1)d$

$a_{150} = 5 + (150-1)11 = 1644$

$\frac{n}{2}(\text{first} + \text{last})$

$\frac{150}{2}(5 + 1644)$

$123,675$