Miscellaneous Vector Concepts

Concept 1: Vectors in the Plane Applications

1. A commercial jet is flying from Miami to Seattle. The jet's velocity with respect to the air is 580 mph, and its bearing is 332°. The wind, at the altitude of the plane, is blowing from the southwest w/ a velocity of 60 mph.

   a. Draw a visual representation, using vectors, of the problem.

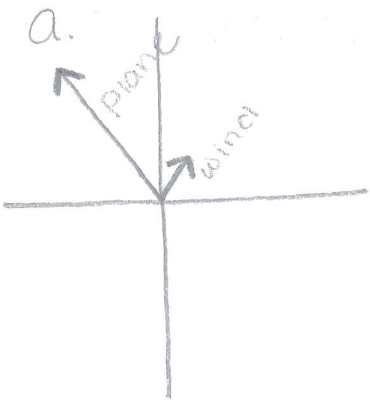
   b. → write a vector in component form that represents the wind.

   c. Write a vector in component form that represents the airspeed of the jet.

   d. What is the speed of the jet w/ respect to the ground?

   e. What is the true direction of the jet?

a.



b. $\langle 60\cos 45°, 60\sin 45° \rangle$

$= \langle 42.426, 42.426 \rangle$

c. $\langle 580\cos 118, 580\sin 118 \rangle$

↓ direction angle!

$= \langle -272.294, 512.109 \rangle$

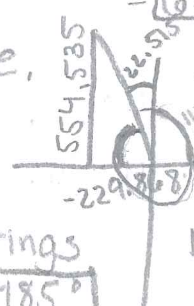
d. ground = air + wind

$\langle -229.868, 554.535 \rangle$

$\sqrt{(-229.868)^2 + (554.535)^2}$

$= \boxed{600.29 \text{ mph}}$

e.



$\tan^{-1}\left(\dfrac{554.535}{-229.868}\right)$

$= 67.485$

True direction = Bearings

$= \boxed{337.485°}$

Direction angle = 112.515°

Concept 2: The dot Product

Notes: The dot product of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$$

★ your final answer is a constant, not a vector!

Example: Let $\mathbf{u} = \langle -1, 3 \rangle$ and $\mathbf{v} = \langle 2, -4 \rangle$. Determine $\mathbf{u} \cdot \mathbf{v}$

$(-1)(2) + (3)(-4)$

$-2 + -12 = \boxed{-14}$

→

1. Let $u = \langle -1, 3 \rangle$, $v = \langle 2, -4 \rangle$, and $w = \langle 1, -2 \rangle$. Determine the following:

dot product

a. $(u \bullet v)w$

b. $u \bullet 2v$

c. $\|u\|$

a. $u \cdot v = (-1)(2) + (3)(-4) = -2 + -12 = -14$

$-14\langle 1, -2 \rangle = \boxed{\langle -14, 28 \rangle}$

b. $2v = \langle 4, -8 \rangle$

$\langle -1, 3 \rangle \cdot \langle 4, -8 \rangle$

$(-1)(4) + (3)(-8) = -4 + -24 = \boxed{-28}$

c. $\sqrt{(-1)^2 + (3)^2} = \boxed{\sqrt{10}}$

Concept 3: Angle Between Vectors

1. Use any method you can think of to determine the angle between $u = \langle 4, 3 \rangle$ and $v = \langle 3, 5 \rangle$. Perhaps you should draw a picture to get started :)

2. Are vectors $u = \langle 2, -3 \rangle$ and $v = \langle 6, 4 \rangle$ orthogonal? (Orthogonal means perpendicular in vector language)

3. Use the formula, below, to try the first to problems again. This should be a lot easier.

If θ is the angle between two nonzero vectors u and v, then

$$\cos\theta = \frac{u \bullet v}{\|u\| \, \|v\|}$$

dot product!

1. $u \cdot v = (4)(3) + (3)(5) = 12 + 15 = 27$

$\|u\| = \sqrt{16 + 9} = 5$

$\|v\| = \sqrt{9 + 25} = \sqrt{34}$

$\frac{27}{(5)(\sqrt{34})} = \cos\theta$

$\cos^{-1}\left(\frac{27}{(5)\sqrt{34}}\right) = \boxed{22.18^\circ}$