

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin \theta = \frac{\sqrt{2}}{2}$

$\frac{3\pi}{4} + 2\pi n$ and $\frac{\pi}{4} + 2\pi n$

2. $\cos \theta = \sin \theta$

$\frac{\pi}{4} + \pi n$

3. $\tan \theta = 1$

$\frac{\pi}{4} + \pi n$

4. $1 + \sin \theta = 2 \cos^2 \theta$

$1 + \sin \theta = 2(1 - \sin^2 \theta)$

$1 + \sin \theta = 2 - 2 \sin^2 \theta$

$-1 + \sin \theta + 2 \sin^2 \theta = 0$

$2 \sin^2 \theta + \sin \theta - 1 = 0$

$(2 \sin \theta - 1)(\sin \theta + 1)$

$2 \sin \theta - 1 = 0$ $\sin \theta + 1 = 0$

$\sin \theta = \frac{1}{2}$ $\sin \theta = -1$

$\frac{5\pi}{6} + 2\pi n, \frac{\pi}{6} + 2\pi n, \frac{3\pi}{2} + 2\pi n$

7. $\sin^2 \theta - 1 = 0$

$\sin^2 \theta = 1$

$\sin \theta = \pm 1$

$\frac{\pi}{2} + \pi n$

5. $2 \cos^2 \theta + \cos \theta = 0$

$\cos \theta (2 \cos \theta + 1) = 0$

$\cos \theta = 0$ $2 \cos \theta + 1 = 0$

$\frac{\pi}{2} + \pi n$ $\frac{2\pi}{3} + 2\pi n$
 $\frac{4\pi}{3} + 2\pi n$

6. $\sin 3\theta = -1$

$\frac{3\pi}{2} + 2\pi n = 3\theta$

$\frac{3\pi}{6} + \frac{2}{3}\pi n = \theta$

Reduce

$\frac{\pi}{2} + \frac{2}{3}\pi n$

9. $2 \sin^2 \theta - \sin \theta - 1 = 0$

$(2 \sin \theta + 1)(\sin \theta - 1)$

$\sin \theta = \frac{1}{2}$ $\sin \theta = 1$

$\frac{7\pi}{6} + 2\pi n$ $\frac{\pi}{2} + 2\pi n$
 $\frac{11\pi}{6} + 2\pi n$

10. $\tan 4\theta = -1$

$4\theta = \frac{3\pi}{4}$ and $\frac{7\pi}{4}$

$\frac{3\pi}{4} + \pi n = 4\theta$

$\frac{3\pi}{16} + \frac{\pi}{4}n$

11. $\tan^2 3x = 3$

$\tan 3x = \pm \sqrt{3}$

$\frac{\pi}{3} + \pi n = 3x$

$\frac{\pi}{9} + \frac{\pi}{3}n$
and
 $\frac{2\pi}{9} + \frac{\pi}{3}n$

12. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$

$\frac{\pi}{4}$ and $\frac{7\pi}{4}$

$\frac{\pi}{4} + 2\pi n$ and $\frac{7\pi}{4} + 2\pi n = \frac{x}{2}$

$x = \frac{\pi}{2} + 4\pi n$ and $\frac{7\pi}{2} + 4\pi n$

Solve for the unknown variable on the interval $0 \leq x < 2\pi$.

1. $4 \cos^2 x - 3 = 0$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$\boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

2. $\sqrt{2} \sin 2x = 1$

$$\sin 2x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$2x = \frac{\pi}{4} \text{ and } 2x = \frac{3\pi}{4}$$

$$\boxed{x = \frac{\pi}{8} \text{ and } \frac{3\pi}{8}}$$

3. $3 \cot^2 x - 1 = 0$

$$\cot^2 x = \frac{1}{3}$$

$$\cot x = \pm \frac{1}{\sqrt{3}}$$

$$\tan x = \pm \sqrt{3}$$

$$\boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

4. $\cos^3 x = \cos x$

$$\cos^3 x - \cos x = 0$$

~~$$\cos x (\cos^2 x - 1)$$~~

$$\cos x = 0 \quad \cos^2 x - 1 = 0$$

$$\cos x = \pm 1$$

$$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi}$$

5. $\sin x - 2 \sin x \cos x = 0$

$$\sin x (1 - 2 \cos x) = 0$$

$$\sin x = 0$$

$$1 - 2 \cos x = 0$$

$$\cos x = \frac{1}{2}$$

$$\boxed{0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}}$$

6. $2 \sin^2 x - \sin x - 3 = 0$

$$(2 \sin x - 3)(\sin x + 1) = 0$$

$$2 \sin x - 3 = 0$$

$$\sin x = \frac{3}{2} \rightarrow \text{NEVER}$$

$$\sin x = -1 = \boxed{\frac{3\pi}{2}}$$

7. $\csc^2 x - \csc x - 2 = 0$

$$(\csc x - 2)(\csc x + 1) = 0$$

$$\csc x = 2 \quad \csc x = -1$$

$$\sin x = \frac{1}{2} \quad \sin x = -1$$

$$\boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$$

8. $\cos^2 x = 1 - \sin x$

$$1 - \sin^2 x - 1 + \sin x = 0$$

$$-\sin^2 x + \sin x = 0$$

$$-(\sin^2 x - \sin x) = 0$$

$$\sin x (\sin x - 1) = 0$$

$$\boxed{0, \pi, \frac{\pi}{2}}$$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(2x) = 0$ on $[0, 2\pi)$.

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

11. $\sin\left(\frac{x}{2}\right) - 1 = 0$ on $[0, 8\pi)$.

— SKIP —