

Winter Break Assignment

Evaluate each limit.

1) $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 5x + 6}$

2) $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\sin(5x)}$

3) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{5x}$

4) $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 + 3}}{2x}$

5) $\lim_{x \rightarrow \infty} (x^3 - 2x^2 - 3)$

6) $\lim_{x \rightarrow -\infty} -e^{\frac{1}{x}}$

Use logarithmic differentiation to differentiate each function with respect to x .

7) $y = 3x^{x^5}$

8) $y = 2x^{x^4}$

9) $y = 5x^{2x}$

10) $y = x^{3x}$

$$11) y = 2x^{x^5}$$

Differentiate each function with respect to x .

$$12) y = \ln \left(\frac{5x^5}{x^2 + 3} \right)^5$$

$$13) y = \ln \left(\frac{4x^5}{x^3 + 3} \right)^3$$

$$14) y = \ln \left(\frac{5x^2}{5x^3 - 4} \right)^4$$

$$15) y = \ln \left(\frac{5x^2}{x^3 + 2} \right)^3$$

$$16) y = \ln \left(\frac{x^4}{2x^3 - 3} \right)^5$$

$$17) y = \log_4 \left(\frac{5x^5}{x^4 - 3} \right)^4$$

$$18) y = \log_2 \left(\frac{4x^3}{x^4 + 5} \right)^3$$

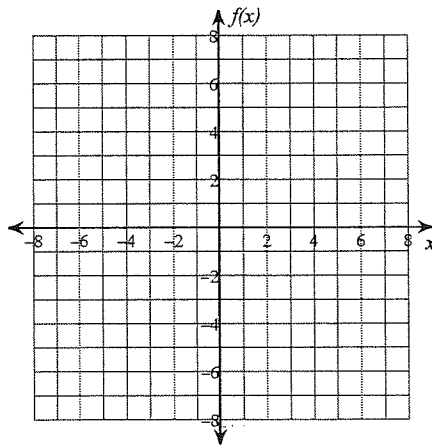
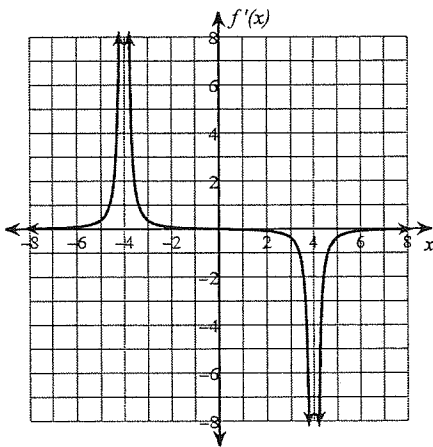
$$19) y = \log_4 \left(\frac{x^3}{3x^4 - 2} \right)^4$$

$$20) y = \log_2 \left(\frac{2x^3}{x^2 - 2} \right)^4$$

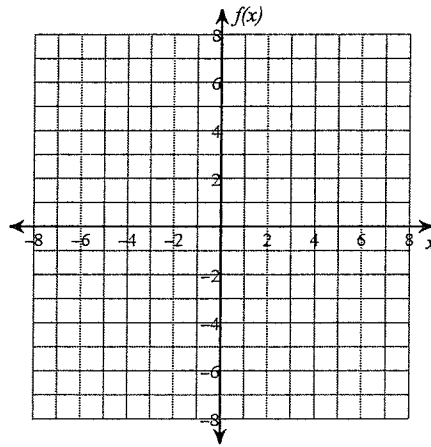
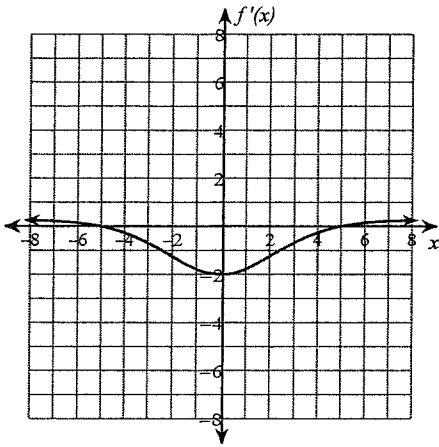
$$21) y = \log_3 \left(\frac{x^2}{4x^3 - 1} \right)^2$$

Given the graph of $f'(x)$, sketch a possible graph of $f(x)$.

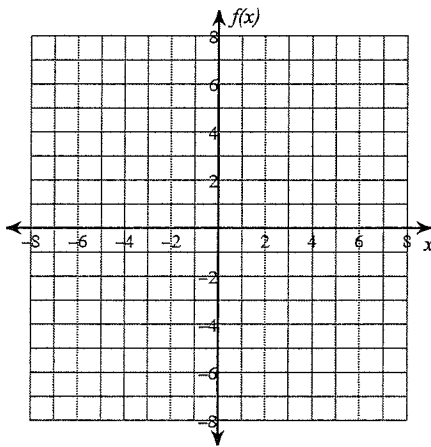
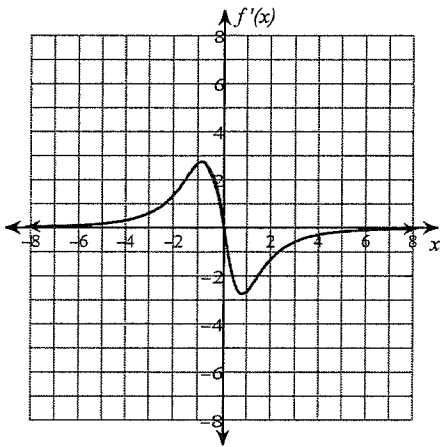
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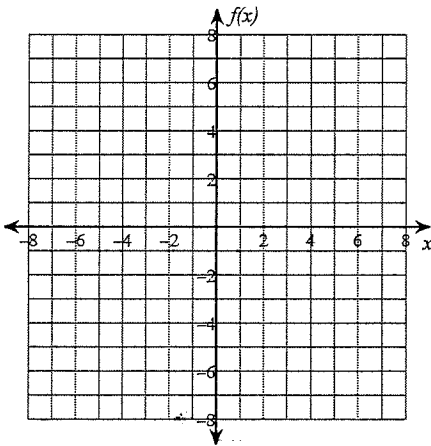
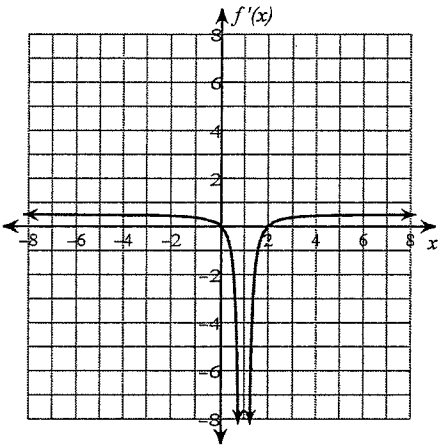
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24)



25)



Solve each optimization problem.

26) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(5, 0)$?

- 27) A graphic designer is asked to create a movie poster with a 50 in² photo surrounded by a 2 in border at the top and bottom and a 1 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?
- 28) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 864 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

For each problem, find the values of c that satisfy the Mean Value Theorem.

29) $y = -\frac{x^2}{2} + 3x + \frac{1}{2}$; $[1, 6]$

30) $y = -x^2 + 4x$; $[-1, 4]$

Other Topics to Study:

Topic 1: Percentage Growth Rate:

1. What grows faster: power functions or exponential functions?
2. Which function has the larger average percentage growth rate for large x 's?
 - a. $\log(x)$
 - b. e^x
 - c. x^2
 - d. x^3

Topic 2: Limiting Values (Limits):

Determine the following: (Hint*** Remember those questions where you had to rank different functions in order of dominance? Those were important! It is important which functions grow faster than others.)

3. $\lim_{x \rightarrow \infty} \frac{4x^7 + 2e^{0.2x}}{2x^{10} + e^{0.2x}}$	4. $\lim_{x \rightarrow -\infty} \frac{4x^7 + 2e^{0.2x}}{2x^{10} + e^{0.2x}}$
5. $\lim_{x \rightarrow \infty} \frac{\text{Log}[x]}{x^2}$	6. $\lim_{x \rightarrow -\infty} \frac{\text{Log}[x]}{x^2}$
7. $\lim_{x \rightarrow \infty} \frac{e^x}{4x^2}$	8. $\lim_{x \rightarrow -\infty} \frac{e^x}{4x^2}$

9. $\lim_{x \rightarrow \infty} \frac{4x^2}{e^x}$

10. $\lim_{x \rightarrow -\infty} \frac{4x^2}{e^x}$

Topic 3: Limit Definition of the Derivative

1.

What is $\lim_{h \rightarrow 0} \frac{8\left(\frac{1}{2} + h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

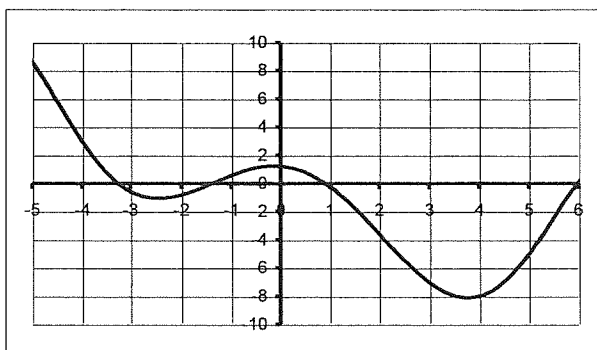
(E) It cannot be determined from the information given.

2.

10. $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$ is

- (A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$ (E) nonexistent

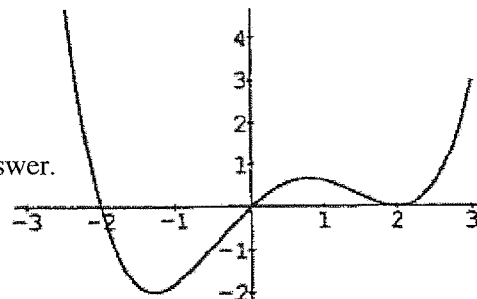
1. A graph of $f'(x)$, the derivative of $f(x)$, is given below.



- On what interval(s) is $f(x)$ increasing? Decreasing? Explain.
- Determine where any relative extrema will occur. Justify your reasoning.

2. The graph at the right is the *derivative* of a function f .

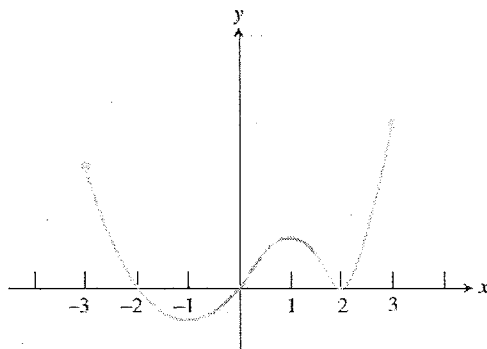
- Find where f is increasing or decreasing. Justify your answer.
- Find all relative maximum(s) or minimum(s). Justify your answer.
- If $f(-3) = 2$, sketch a possible graph of f on the same axes.



3. Sketch a graph of the function whose *derivative* satisfies the properties given in the table below.

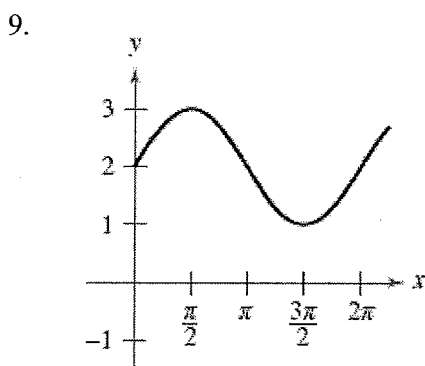
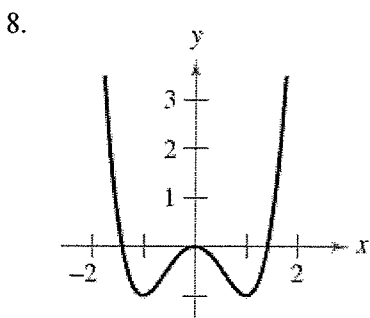
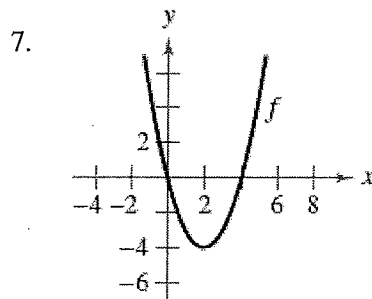
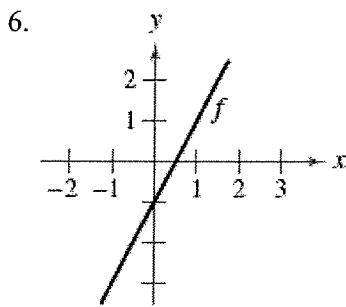
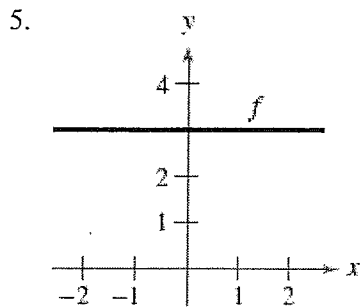
x	$(-\infty, -1)$	-1	$(-1, 1)$	1	$(1, 3)$	3	$(3, \infty)$
$f'(x)$	positive	0	negative	0	positive	0	negative

4. The accompanying figure shows the graph of the derivative of a function f . The domain of f is the closed interval $[-3, 3]$.



- Identify and classify the x -coordinate of each critical value. Justify your answers.
- Determine the interval(s) on which f is increasing. Justify.
- Determine the interval(s) on which f is concave up and concave down. Justify your answers.

The graph of f is given below. Sketch a possible graph of f' and f'' .



The graph of f' is given below. Sketch a possible graph of f .

