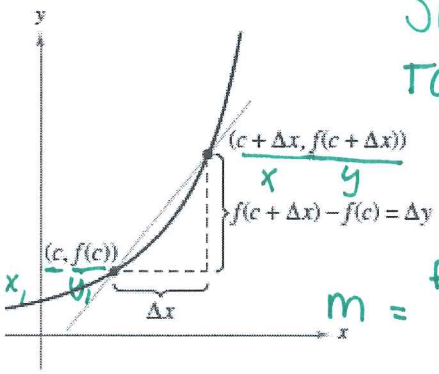
Notes 10/11

Lesson 3 Lecture Day 1: Continuity and Differentiability
Homework: 5-41 odd (problems attached)

The Slope of the Secant Line:



Secant line: touches in 2 points
Tangent line: touches @ 1 point

Slope of secant line = $\frac{y - y_1}{x - x_1}$

$m = \frac{f(c+\Delta x) - f(c)}{c + \Delta x - c} = \frac{f(c+\Delta x) - f(c)}{\Delta x}$

The Slope of the Tangent Line: → as the distance between the 2 points in the secant line gets smaller and smaller (approaching zero), we get a tangent line.

Definition of Tangent Line with Slope m

If f is defined on an open interval containing c, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through $(c, f(c))$ with slope m is the **tangent line** to the graph of f at the point $(c, f(c))$.

EXAMPLE 2 **Tangent Lines to the Graph of a Nonlinear Function**

Find the slopes of the tangent lines to the graph of $f(x) = x^2 + 1$ at the points $(0, 1)$ and $(-1, 2)$, as shown in Figure 2.6.

$$\lim_{\Delta x \to 0} \frac{(c+\Delta x)^2 + 1 - (c^2 + 1)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{c^2 + 2c\Delta x + \Delta x^2 + 1 - c^2 - 1}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2c\Delta x + \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2c + \Delta x)}{\Delta x} = 2c + \Delta x$$

$$\lim_{\Delta x \to 0} 2c + \Delta x = 2c$$

The Derivative of a Function

gives you a function that tells you the slope of the tangent line @ any given point

Definition of the Derivative of a Function

The derivative of f at x is

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

↙ notation for the derivative

provided the limit exists. For all x for which this limit exists, f' is a function of x.

Example 2:

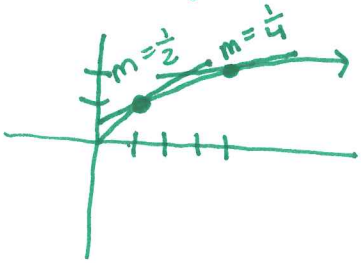
Find $f'(x)$ for $f(x) = \sqrt{x}$. Then find the slopes of the graph of f at the points $(1, 1)$ and $(4, 2)$. Discuss the behavior of f at $(0, 0)$.

$$\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{x+\Delta x - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}} = f'(x)} = \text{the derivative}$$

@ $(1,1) \to \frac{1}{2\sqrt{1}} = \frac{1}{2}$

@ $(4,2) \to \frac{1}{2\sqrt{4}} = \frac{1}{4}$



Example 3:

Find the derivative with respect to t for the function $y = 2/t$.

↗ should use "t" instead of "x"

$$\lim_{\Delta x \to 0} \frac{\frac{2}{x+\Delta x} - \frac{2}{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{2x}{x(x+\Delta x)} - \frac{2(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2x - 2x - 2\Delta x}{x(x+\Delta x)}}{\Delta x} = \frac{-2\Delta x}{x(x+\Delta x)} \cdot \frac{1}{\Delta x} = \lim_{\Delta x \to 0} \frac{-2}{x(x+\Delta x)} = \boxed{\frac{-2}{x^2} = f'(x)}$$

Homework ↓

Finding the Slope of a Tangent Line **In Exercises 5–10, find the slope of the tangent line to the graph of the function at the given point.**

5. $f(x) = 3 - 5x, \quad (-1, 8)$
6. $g(x) = \frac{3}{2}x + 1, \quad (-2, -2)$
7. $g(x) = x^2 - 9, \quad (2, -5)$
8. $f(x) = 5 - x^2, \quad (3, -4)$
9. $f(t) = 3t - t^2, \quad (0, 0)$
10. $h(t) = t^2 + 4t, \quad (1, 5)$

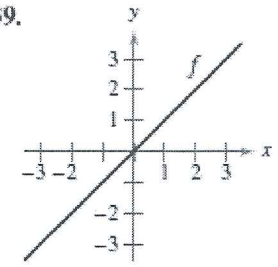
Finding the Derivative by the Limit Process **In Exercises 11–24, find the derivative of the function by the limit process.**

11. $f(x) = 7$
12. $g(x) = -3$
13. $f(x) = -10x$
14. $f(x) = 7x - 3$
15. $h(s) = 3 + \frac{2}{3}s$
16. $f(x) = 5 - \frac{2}{3}x$
17. $f(x) = x^2 + x - 3$
18. $f(x) = x^2 - 5$
19. $f(x) = x^3 - 12x$
20. $f(x) = x^3 + x^2$

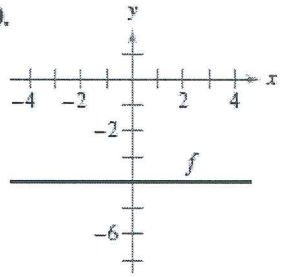
WRITING ABOUT CONCEPTS

Sketching a Derivative **In Exercises 39–44, sketch the graph of f'. Explain how you found your answer.**

39.



40.



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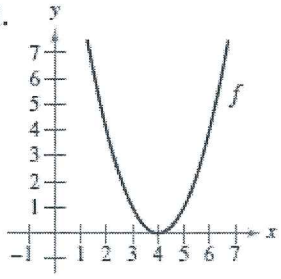

42.