8.3 Geometric Sequences and Series

What is a geometric sequence? A sequence is geometric when the ratios of consecutive terms are the same. In other words, to get from one term to the next, you *multiply* by the same number. That number is the **ratio**.

How do I find the ratio? Divide two consecutive terms. For example: If a sequence is geometric, and $a_1 = 1$ and $a_2 = 4$, then the ratio is $\frac{4}{1} = 4$.

What is an example of a geometric sequence? Geometric sequence: $2, \ 4, \ 8, \ 16 \ldots.$

The common ratio is $\frac{4}{2} = 2$. This means you multiply by two to get the next term.

How do I find the nth term of a geometric sequence?

$$a_n = a_1 r^{n-1}$$

r= ratio

How do I find the sum of a finite geometric sequence?

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1 \left(\frac{1-r^n}{1-r} \right)$$

r= ratio

How do I find the sum of an infinite geometric sequence?

$$S = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}$$

r= ratio

*A finite sum only exists when r<1

I Do: ☆Calculator OK   You Do:

I Do	You Do
1a. Write the first five terms of the geometric sequence whose first term is $a_1 = 3$ and whose common ratio is $r = 2$ $a_1 = 3$ $a_2 = 6$ $a_3 = 12$ $a_4 = 24$ $a_5 = 48$	**1b.** Write the first five terms of the geometric sequence whose first term is $a_1 = 2$ and $r = \frac{1}{3}$
2a. Find the 15th term of the geometric sequence whose first term is $a_1 = 20$ and whose common ratio is $r = 1.05$ $a_n = a_1 r^{n-1}$ $a_{15} = 20(1.05)^{15-1}$ $= 39.60$	**2b.** Find the 12th term of the geometric sequence whose first term is $a_1 = -\frac{1}{128}$ and whose common ratio is $r = 2$

3a. Find a formula for the nth term of the following geometric sequence. What is the ninth term? 5, 15, 45.... $$a_n = 5(3)^{n-1}$$ $$a_9 = 5(3)^{9-1}$$ $$= 32,805$$	**3b. Find a formula for the nth term of the following geometric sequence. What is the 22nd term?** 4, 8, 16.....
4a. The 4th term of a geometric sequence is 125, and the 10th term is $\frac{125}{64}$. Find the 14th term. Assume at the terms of the sequence are positive. $a_{10} = a_4 r^6$ (6 terms between a_4 and a_{16}) $\frac{125}{64} = 125 r^6$ $\frac{1}{64} = r^6 \Rightarrow r = \frac{1}{2}$ 4 terms b/w the 10th & 14th term $$a_{14} = \frac{125}{64}\left(\frac{1}{2}\right)^4 = \frac{125}{1024}$$	**4b. The second term of a geometric sequence is -18 and the fifth term is $\frac{2}{3}$, find the 6th term.**
5a. Find the following sum. $$\sum_{n=1}^{12} 4(0.3)^n \quad \rightarrow \text{finite}$$ $$a_1\left(\frac{1-r^n}{1-r}\right)$$ $$a_1 = 4(0.3)^1 = 4(0.3)$$ $$4(0.3)\left(\frac{1-0.3^{12}}{1-0.3}\right) = 1.71$$	**5b. Find the following sum.** $$\sum_{n=0}^{20} 3\left(\tfrac{3}{2}\right)^n$$
6a. Find the following sum. $$\sum_{n=0}^{\infty} 10\left(\tfrac{4}{5}\right)^n$$ $a_1 = 10$ (plug in zero, thats where the sum starts) $$\frac{a_1}{1-r} = \frac{10}{1-\frac{4}{5}} = \frac{10}{\frac{1}{5}} = 50$$	**6b. Find the following sum.** $$\sum_{n=1}^{\infty} 8\left(\tfrac{5}{3}\right)^{n-1}$$