Derivatives Practice Solutions

21. $y = \frac{1}{\sqrt{3x+5}} = (3x+5)^{-1/2} = -1/2\,(3x+5)^{-3/2} \cdot 3 = \frac{-3}{2}(3x+5)^{-3/2}$

$$= \boxed{\frac{-3}{2(3x+5)^{3/2}}}$$

24. $f(x)\,x(2x-5)^3 = (x)(3(2x-5)^2)(2) + (2x-5)^3(1)$

$$= 6x(2x-5)^2 + (2x-5)^3$$

$$= (2x-5)^2(6x+2x-5) = \boxed{(2x-5)^2(8x-5)}$$

29. $g(x) = \left(\frac{x+5}{x^2+2}\right)^2$

$$g'(x) = 2\left(\frac{x+5}{x^2+2}\right)\left(\frac{(x^2+2)(1)-(x+5)(2x)}{(x^2+2)^2}\right) = \boxed{\frac{2(x+5)(2-10x-x^2)}{(x^2+2)^3}}$$

30. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2 = 2\left(\frac{t^2}{t^3+2}\right)\left(\frac{(t^3+2)(2t)-(t^2)(3t^2)}{(t^3+2)^2}\right) =$

$$\frac{2(t^2)(2t^4+4t-3t^4)}{(t^3-2)^3} = \frac{2t^2(-t^4+4t)}{(t^3-2)^3} = \boxed{\frac{2t^3(-t^3+4}{(t^3-2)^3}}$$

33. $f(x) = ((x^2+3)^5+x)^2$

$$f'(x) = 2((x^2+3)^5+x)\,(5(x^2+3)^4(2x)+1)$$

FOIL

$$2\left[5(x^2+3)^9(2x) + (x^2+3)^5 + 2x^2(5(x^2+3)^4) + x\right]$$

$$= \boxed{20x(x^2+3)^9 + 2(x^2+3)^5 + 20x^2(x^2+3)^4 + 2x}$$

43. $y = \cos 4x = \cos(4x)$

$$y' = -\sin(4x)\cdot 4 = \boxed{-4\sin 4x}$$

49. $\sin 2x \cos 2x = f(x)$

$$f'(x) = \sin 2x \cdot (-2\sin 2x) + \cos 2x(2\cos 2$$

$$\boxed{-2\sin^2(2x) + 2\cos^2 2x}$$

53. $y = 4\sec^2 x$

$y' = 4(2)\sec x \cdot \sec x \tan x = \boxed{8\sec^2 x \tan x}$

59. $f(t) = 3\sec^2(\pi t - 1)$

$= 3\sec(\pi t - 1)^2$

$f'(t) = 6\sec(\pi t - 1) \cdot \sec(\pi t - 1)\tan(\pi t - 1) \cdot \pi$

$= \boxed{6\pi \sec^2(\pi t - 1)(\tan \pi t - 1)}$

61. $y = \sqrt{x} + \frac{1}{4}\sin(2x)^2 = \sqrt{x} + \frac{1}{4}\sin(4x^2)$

$y' = \frac{1}{2}x^{-1/2} + \frac{1}{4}\cos(4x^2) \cdot 8x$

$= \boxed{\frac{1}{2\sqrt{x}} + 2x\cos(4x^2)}$

62. $y = \sin \sqrt[3]{x} + \sqrt[3]{\sin x}$

$y = \sin(x)^{1/3} + (\sin x)^{1/3}$

$y' = \cos \sqrt[3]{x} \cdot \frac{1}{3}x^{-2/3} + \frac{1}{3}\sin x^{-2/3} \cdot \cos x$

$\boxed{\frac{1}{3}x^{-2/3}\cos\sqrt[3]{x} + \frac{1}{3}\cos x(\sin x^{-2/3})}$

73. $y = \sqrt{2x^2 - 7}$ $(4, 5)$

$\frac{1}{2}(2x^2 - 7)^{-1/2} \cdot 4x$

$\frac{4x}{2\sqrt{2x^2-7}}$ $\frac{4(4)}{2\sqrt{32-7}} = \frac{16}{2(25)} = \frac{16}{10} = \frac{8}{5}$

$\boxed{y - 5 = \frac{8}{5}(x - 4)}$

76. $f(x) = (9 - x^2)^{2/3}$ $(1, 4)$

$\frac{2}{3}(9 - x^2)^{-1/3} \cdot -2x$

$= \frac{-4x}{3\sqrt[3]{9-x^2}} = \frac{-4}{3\sqrt[3]{9-1}} = \frac{-4}{3(2)} = \frac{-4}{6} = \frac{-2}{3}$

$\boxed{y - 4 = \frac{-2}{3}(x - 1)}$

79. $f(x) = \tan^2 x$

$f'(x) = 2\tan x \sec^2 x$

$f'(\frac{\pi}{4}) = 2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sec\frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$= 2(1)(2) = 4$

$\boxed{y - 1 = 4(x - \frac{\pi}{4})}$

83. $y = 2\cos x + \sin 2x$

$(-2\sin x + 2\cos 2x) = 0$

$-2\sin x + 2(1 - 2\sin^2 x)$

$-2\sin x + 2 - 4\sin^2 x$

$-2(2\sin^2 x + 1\sin x - 1)$

$-2(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ $\sin x + 1 = 0$

$\boxed{x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$

86. $y = 6(x^3 + 4)^3$

$y' = 18(x^3 + 4)^2 \cdot 3x^2$

$y' = 54x^2(x^3 + 4)$ should simplify

$\boxed{y'' = 54x^2(3x^2) + (x^3+4)(108x)}$

87. $y = \frac{1}{x-6} = (x-6)^{-1}$

$y' = -1(x-6)^{-2}$

$\boxed{y'' = 2(x-6)^{-3}}$