Chapter 5 Review Solutions

Section #1

1. $\dfrac{1}{\tan^2 x + 1} = \dfrac{1}{\sec^2 x} = \boxed{\cos^2 x}$

2. $\csc^2 x(1 - \cos^2 x)$

$\csc^2 x(\sin^2 x) = \boxed{1}$

3. $\sin x(\csc x - \sin x)$

$\sin x\left(\dfrac{1}{\sin x} - \sin x\right)$

$\dfrac{\sin x}{\sin x} - \sin^2 x = 1 - \sin^2 x = \boxed{\cos^2 x}$

Section #2

1. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$

$\dfrac{-\csc x}{\sec x} = \dfrac{\frac{-1}{\sin x}}{\frac{1}{\cos x}} = -\dfrac{\cos x}{\sin x}$ ✓

2. $\cot^2 x - \cos^2 x = \cot^2 x \cos^2 x$

$\dfrac{\cos^2 x}{\sin^2 x} - \cos^2 x$

$\dfrac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x} = \dfrac{\cos^2 x(1 - \sin^2 x)}{\text{2in}^2 x} = \dfrac{\cos^2 x(\cos^2 x)}{\sin^2 x}$

$= \cot^2 x \cos^2 x$ ✓

Section # 3

1. $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\pm\sqrt{3}}{2}$

$\boxed{\cancel{\frac{\pi}{6} + 2\pi n \text{ and } \frac{11\pi}{6} + 2\pi n}}$

$\frac{\pi}{6} + \pi n$ and $\frac{5\pi}{6} + \pi n$

2. $2\sin^2 x - 3\sin x = -1$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1)$

$2\sin x - 1 = 0 \qquad \sin x - 1 = 0$

$\sin x = \frac{1}{2} \qquad \sin x = 1$

$\boxed{\frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{\pi}{2}}$

3. $\sqrt{3}\tan 3x = 0$

$\tan 3x = 0$

* change to general solution!

$3x = 0 + 2\pi n$

$\boxed{x = 0 + \frac{2}{3}\pi n}$ or just $\frac{2}{3}\pi n$

Section #4

1. $\frac{31\pi}{12} = \frac{11\pi}{6} + \frac{3\pi}{4}$

$\sin\left(\frac{31\pi}{12}\right) = \sin\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right) = \sin\frac{11\pi}{6}\cos\frac{3\pi}{4} + \sin\frac{3\pi}{4}\cos\frac{11\pi}{6}$

$= \left(\frac{-1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

$\cos\left(\frac{31\pi}{12}\right) = \cos\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right) = \cos\left(\frac{11\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{11\pi}{6}\right)\sin\left(\frac{3\pi}{4}\right)$

$\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

$\tan\left(\frac{31\pi}{12}\right) = \dfrac{\frac{\sqrt{2}+\sqrt{6}}{4}}{\frac{-\sqrt{6}+\sqrt{2}}{4}} = \frac{\sqrt{2}+\sqrt{6}}{\cancel{4}} \cdot \frac{\cancel{4}}{-\sqrt{6}+\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}} \cdot \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}$

$= \frac{4+2\sqrt{12}}{4-6} = \frac{4+2\sqrt{12}}{-2} = \frac{\cancel{2}(2+\sqrt{12})}{-\cancel{2}} = -2-\sqrt{12}$

Section #4 continued

2. $\sin(x+\frac{\pi}{2}) - \sin(x-\frac{\pi}{2}) = \sqrt{2}$

$\sin x \cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos x$

$\sin x(0) + (1)\cos x$

$\cos x - \left[\sin x\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\cos x\right]$

$\cos x - \left[\sin x(0) - (1)\cos x\right] = \sqrt{2}$

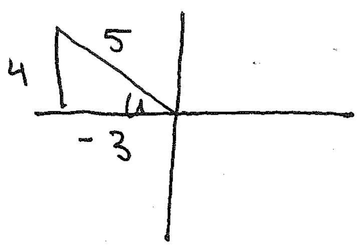
$\cos x + \cos x = \sqrt{2}$

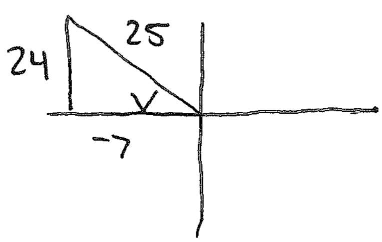
$2\cos x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4} \text{ and } \frac{7\pi}{4}}$

3. ★ Both u and v in Q2



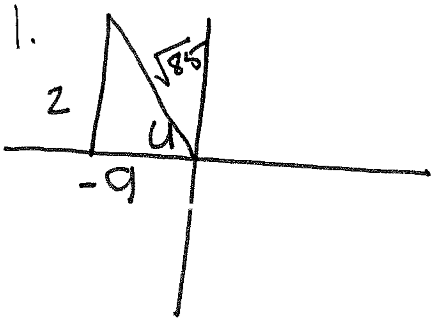


$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\left(\frac{-4}{3}\right) + \left(\frac{-24}{7}\right)}{1 - \left(\frac{-4}{3}\right)\left(\frac{-24}{7}\right)} = \frac{\frac{-28}{21} - \frac{72}{21}}{1 - \frac{96}{21}}$$

$$= \frac{\frac{-100}{21}}{\frac{-75}{21}} = \boxed{\frac{4}{3}}$$

Section #5

1.



$$2^2+(-9)^2=c^2$$
$$4+81=c^2$$
$$85=c^2$$

$$\sin 2u = 2\sin u\cos u = 2\left(\frac{2}{\sqrt{85}}\right)\left(\frac{-9}{\sqrt{85}}\right) = \boxed{\frac{-36}{85}}$$

$$\cos 2u = 2\cos^2 u - 1 = 2\left(\frac{-9}{\sqrt{85}}\right)^2 - 1 = 2\left(\frac{81}{85}\right) - 1$$
$$= \frac{162}{85} - \frac{85}{85} = \boxed{\frac{77}{85}}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u} = \frac{2\left(\frac{-2}{9}\right)}{1-\left(\frac{-2}{9}\right)^2} = \frac{\frac{-4}{9}}{1-\frac{4}{81}} = \frac{\frac{-4}{9}}{\frac{77}{81}} = \frac{-4}{9}\cdot\frac{81}{77}$$
$$= \frac{-324}{693} = \boxed{\frac{-36}{77}}$$

2. $\frac{7\pi}{8}\cdot 2 = \frac{14\pi}{8} = \frac{7\pi}{4}$

$$\sin\left(\frac{\frac{7\pi}{4}}{2}\right) = +\sqrt{\frac{1-\cos\frac{7\pi}{4}}{2}} = +\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = +\sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}}$$

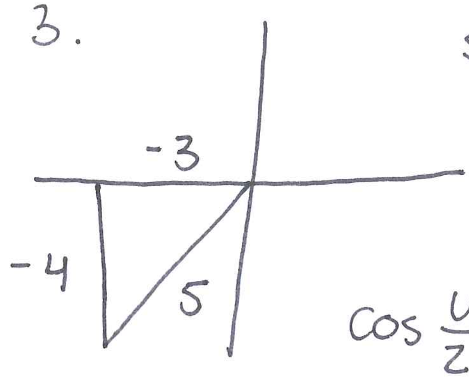
$$\cos\left(\frac{\frac{7\pi}{4}}{2}\right) = -\sqrt{\frac{1+\cos\frac{7\pi}{4}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{-\frac{\sqrt{2+\sqrt{2}}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$

$$\tan\left(\frac{\frac{7\pi}{4}}{2}\right) = \frac{1-\cos\frac{7\pi}{4}}{\sin\frac{7\pi}{4}} = \frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\cancel{2}}\cdot\frac{-\cancel{2}}{\sqrt{2}} = \frac{-2+\sqrt{2}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{-\cancel{2}\sqrt{2}+\cancel{2}}{\cancel{2}}$$
$$= \boxed{-\sqrt{2}+1}$$

Section #5 continued

3.



so if $\pi < u < \frac{3\pi}{2}$

then $\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4}$ where sine is positive and cosine is negative

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\left(\frac{-3}{5}\right)}{2}} = -\sqrt{\frac{5-3}{5}\cdot\frac{1}{2}} = -\sqrt{\frac{1}{5}} = \boxed{\frac{-\sqrt{5}}{5}}$$

$$\sin\frac{u}{2} = \sqrt{\frac{1-\left(\frac{-3}{5}\right)}{2}} = \sqrt{\frac{8}{5}\cdot\frac{1}{2}} = \sqrt{\frac{8}{10}} = \boxed{\frac{2\sqrt{5}}{5}}$$

$$\tan\frac{u}{2} = \frac{2\sqrt{5}}{\cancel{5}}\cdot\frac{\cancel{5}}{-\sqrt{5}} = \frac{2\sqrt{5}}{-\sqrt{5}} = \boxed{-2}$$

4. $\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0 \qquad \tan x = -1$

$$\boxed{0,\ \pi,\ \frac{3\pi}{4},\ \frac{7\pi}{4}}$$