Solutions

Expanding / Condensing Logarithmic Functions Review

Expanding Logarithmic Expressions→ Use the properties of logarithms to expand the expression as a sum, difference, and/or constant multiple of logarithms.

1. $\log_5 5x^2$
2. $\ln\frac{x+3}{xy}$
3. $\log_4 16xy^2$
4. $\ln\frac{xy^5}{\sqrt{z}}$

Condensing Logarithmic Expressions→ Condense the expression to the logarithm of a single quantity.

1. $\log_2 9+\log_2 x$
2. $\log_6 y-2\log_6 z$
3. $\frac{1}{2}\ln(2x-1)-2\ln(x+1)$
4. $3[\ln x-2\ln(x^2+1)]+2\ln 5$

Expanding:

1. $\log_5 5x^2 = \log_5 5 + \log_5 x^2 = 1 + \log_5 x^2 =$ $\boxed{1+2\log_5 x}$
2. $\ln\frac{x+3}{xy} = \ln(x+3)-(\ln x+\ln y)$
3. $\log_4 16xy^2 = \log_4 16 + \log_4 x + 2\log_4 y = 2+\log_4 x+2\log_4 y$
4. $\ln\frac{xy^5}{\sqrt{z}} = \ln x + 5\ln y - \frac{1}{2}\ln z$

Condensing:

1. $\log_2 9+\log_2 x = \log_2 9x$
2. $\log_6 y - 2\log_6 z = \log_6 \frac{y}{\sqrt{z}}$
3. $\frac{1}{2}\ln(2x-1)-2\ln(x+1) = \ln\frac{\sqrt{2x-1}}{(x+1)^2}$
4. $\ln\left[25\left(\frac{x}{x^2+1}\right)^3\right]$

SOLVING EXPONENTIAL/LOGARITHMIC FUNCTIONS REVIEW

Solving an Exponential Equation→ Solve the exponential equation algebraically. Round your result to 3 decimal places.

1. $3e^{-5x} = 132$
2. $2e^{x-3} - 1 = 4$
3. $-e^{x/2} + 1 = \frac{1}{2}$
4. $2(12^x) = 190$
5. $-4(5^x) = -68$
6. $e^{2x} - 6e^x + 8 = 0$

Solving a Logarithmic Equation→ Solve the logarithmic equation algebraically. Round your result to 3 decimal places.

1. $\ln 3x = 6.4$
2. $\ln x - \ln 5 = 2$
3. $\ln\sqrt{x+1} = 2$
4. $\log_4(x-1) = \log_4(x-2) - \log_4(x+2)$
5. $\log_{10}(1-x) = -1$
6. $\log_{10}(-x-4) = 2$

Solving exponential:

1. $3e^{-5x} = 132$

$e^{-5x} = 44$

$-5x = \ln 44$

$x = \frac{\ln 44}{-5} = -.757$

2. $2e^{x-3} - 1 = 4$

$e^{x-3} = 5/2$

$x - 3 = \ln 5/2$

$x = \ln 5/2 + 3 = 3.916$

3. $-e^{x/2} + 1 = 1/2$

$e^{x/2} = +3/2$

$\frac{x}{2} = \ln +3/2$

~~No solution~~

$x = .81$

4. $2(12^x) = 190$

$12^x = 95$

$x = \log_{12} 95$

$= 1.833$

~~Both extraneous~~

5. $-4(5^x) = -68$

$5^x = 17$

$x = \log_5 17 = 1.76$

6. $e^{2x} - 6e^x + 8 = 0$

$(e^x)^2 - (6e^x)^1 + 8 = 0$

$(e^x - 4)(e^x - 2)$

$e^x = 4 \quad e^x = 2$

$x = \ln 4 \quad x = \ln 2$

$x = 1.386 \quad x = .693$

4. $x - 1 = \frac{x-2}{x+2}$

$(x-1)(x+2) = x - 2$

$x^2 + x - 2 = x - 2$

$x^2 = 0 \quad x = 0$, extraneous

Solving Logs:

1. $\ln 3x = 6.4$

$3x = e^{6.4}$

$x = \frac{e^{6.4}}{3} = 200.615$

2. $\ln x - \ln 5 = 2$

$\ln x - 1.6 = 2$

$\ln x = 3.6$

$x = e^{3.6}$

$x = 36.598$

3. $\ln\sqrt{x+1} = 2$

$\frac{1}{2}\ln(x+1) = 2$

$\ln(x+1) = 4$

$x + 1 = e^4$

$x = e^4 - 1$

$x = 53.598$

5. $\log_{10}(1-x) = -1$

$1 - x = 10^{-1}$

$1 - x = \frac{1}{10}$

$x = .9$

6. $-x - 4 = 10^2$

$-x - 4 = 100$

$-x = 104$

$x = -104$

EXTRANEOUS SOLUTIONS REVIEW

Solving a logarithmic equation and checking for extraneous solutions→ Solve the logarithmic equation algebraically. Round your result to 3 decimal places. Check for extraneous solutions.

1. $log5x + log(x-1) = 2$

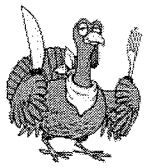
2. $log(x+2) + log(x-1) = 1$

3. $ln(5x+1) = ln(3x+7)$

$2.77 = 2.77$ ✓

4. $ln(x) + ln(x-1) = 1$

5. $3log_2 x = 15$



1. $\log[5x(x-1)] = 2$

$\log(5x^2 - 5) = 2$

$5x^2 - 5 = 10^2$

$5x^2 - 5 = 100$

$5x^2 = 105$

$x^2 = 21$

$x = \pm\sqrt{21}$

$x = \sqrt{21}$

$-\sqrt{21}$ = extraneous

2. $\log(x+2) + \log(x-1) = 1$

$\log(x^2 + x - 2) = 1$

$x^2 + x - 2 = 10$

$x^2 + x - 12 = 0$

$(x+4)(x-3)$

$x = -4 \quad x = 3$

↓

extraneous

3. $5x+1 = 3x+7$

$2x - 6 = 0$

$2x = 6$

$x = 3$ ✓

4. $\ln x(x-1) = 1$

$\ln x^2 - x = 1$

$x^2 - x = e$

(use calculator)

$x^2 - x - e = 0$

$x = -1.222 \quad x = 2.222$

↓

extraneous

5. $3\log_2 x = 15$

$\log_2 x = \frac{15}{2}$

$x = 2^{15/2}$

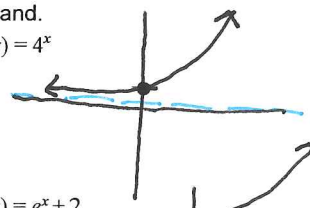
$x = 181.019$ ✓

Solutions

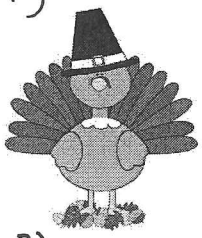
Graphing Exponential/ Logarithmic Functions Review

Graphing Exponential Functions→ Find the ***range***, ***horizontal asymptote***, and ***y-intercept*** of the exponential function, and sketch the graph by hand.

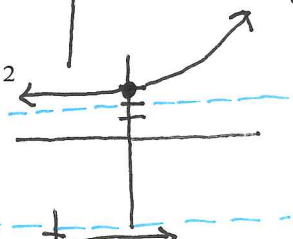
1. $f(x) = 4^x$



Domain: $(-\infty, \infty)$ y-int $= (0,1)$
Range: $(0, \infty)$
H.A: $y = 0$

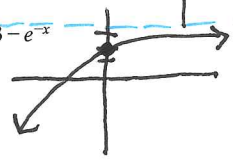


2. $f(x) = e^x + 2$



Domain: $(-\infty, \infty)$
Range: $(2, \infty)$ y-int $= (0,3)$
H.A: $y = 2$

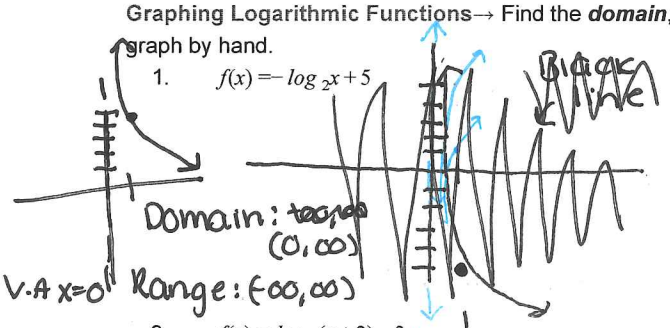
3. $f(x) = 3 - e^{-x}$



Domain: $(-\infty, \infty)$
Range: $(-\infty, 3)$ H.A. $= y = 3$ y-int $= (0,2)$

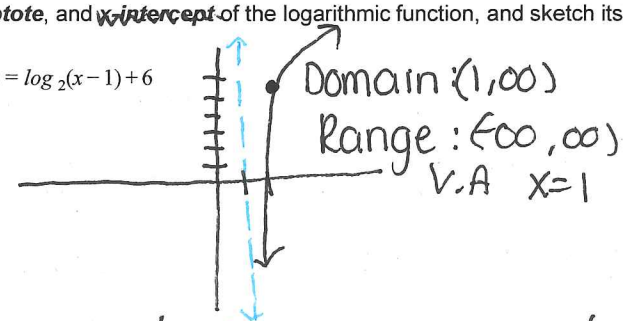
Graphing Logarithmic Functions→ Find the ***domain***, ***vertical asymptote***, and ~~***x-intercept***~~ of the logarithmic function, and sketch its graph by hand.

1. $f(x) = -\log_2 x + 5$



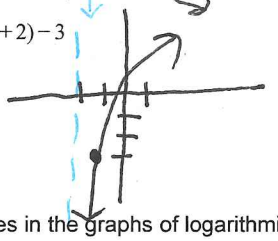
Domain: ~~$(-\infty, \infty)$~~ $(0, \infty)$
V.A $x = 0$ Range: $(-\infty, \infty)$

2. $f(x) = \log_2(x-1) + 6$



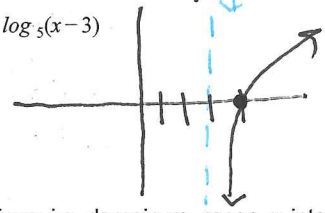
Domain: $(1, \infty)$
Range: $(-\infty, \infty)$
V.A $x = 1$

3. $f(x) = \log_5(x+2) - 3$



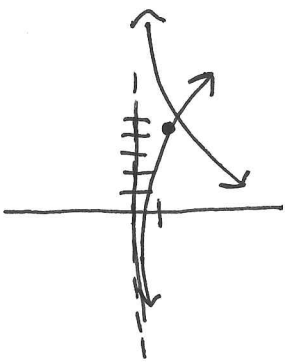
Domain: $(-2, \infty)$
Range: $(-\infty, \infty)$
V.A $x = -2$

4. $f(x) = \log_5(x-3)$



Domain: $(3, \infty)$
Range: $(-\infty, \infty)$
V.A $= x = 3$

**Notice the differences in the graphs of logarithmic and exponential functions: i.e. domain vs. range, x-intercept vs. y-intercept. Why are there these differences?



Rewriting/Simplifying Logarithms

Rewriting Equations→ Write the logarithmic equation in exponential form or write the exponential equation in logarithmic form.

1. $\log_5 125 = 3$

$5^3 = 125$

2. $4^3 = 64$

$\log_4 64 = 3$

3. $\log_{10} \frac{1}{100} = -2$

$10^{-2} = \frac{1}{100}$

4. $12^{-1} = \frac{1}{12}$

$\log_{12} \frac{1}{12} = -1$

Simplifying a Logarithm→ Use the properties of logarithms to rewrite and simplify the logarithmic expression.

1. $\ln\sqrt{e^5}$

$\ln(e^5)^{1/2} = \ln e^{5/2}$

$= 5/2 \ln e = 5/2$

2. $\log_3(9^2 \cdot 2^4)$

$\log_3 9^2 + \log_3 2^4$

$2 + \log_3 2^4$

$= 2 + 4\log_3 2$