AP® Review Questions for Chapter 2

1. (no calculator)
Let $f(x) = (x^2 - 3)^4$. $4(x^2-3)^3(2x)$
(a) Write an equation of the line tangent to the graph of f at $x = 2$. $4(1)(4)=16$ $y-1=16(x-1)$
(b) Find the values of x for which the graph of f has a horizontal tangent. (minimum/max values)
(c) Find $f''(x)$. $f'(x) = \underbrace{8x}\underbrace{(x^2-3)^3}$
$8x\cdot 3(x^2-3)^2\cdot 2x + (x^2-3)^3\cdot 8$
(on back)

2. (no calculator)
Let $f(x) = \sqrt{4x-3}$ and $g(x) = \dfrac{f(x)}{x}$.
(a) What is the slope of the graph of f at $x = 3$? Show the work that leads to your answer.
(b) Write an equation of the line tangent to the graph of g at $x = 3$.
(c) What is the slope of the line normal to the graph of g at $x = 3$?

3. (no calculator)
Evaluate each limit analytically.
(*Note: Finding the answer should not involve a lengthy algebraic process.*)
(a) $\lim\limits_{h\to 0} \dfrac{\sin(x+h) - \sin x}{h}$
(b) $\lim\limits_{h\to 0} \dfrac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$
(c) $\lim\limits_{h\to 0} \dfrac{\sqrt{16+h} - 4}{h}$
(d) $\lim\limits_{h\to 0} \dfrac{\dfrac{1}{5+h} - \dfrac{1}{5}}{h}$

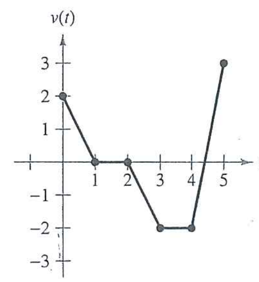
4. Given:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	−3	1	5	−2
5	4	7	−1	2

(a) If $h(x) = \dfrac{f(x)}{g(x)}$, find $h'(2)$.
(b) If $j(x) = f(g(x))$, find $j'(2)$.
(c) If $k(x) = \sqrt{f(x)}$, find $k'(5)$.

5. (no calculator)
Given: $f(x) = x^2$
(a) Find the slope of the normal line to the graph of f at $x = -3$. perpendicular
$f'(x)=2x$ $2(-3)=-6$
(b) Two lines passing through the point (3, 8) will be tangent to the graph of f. Find an equation for each of these lines.
SKIP
$y-9=\frac{1}{6}(x+3)$

6. The accompanying diagram shows the graph of the velocity in $\dfrac{\text{ft}}{\text{sec}}$ for a particle moving along the line $x = 4$.



(a) During which time interval is the particle:
(i) moving upward? When velocity is positive $[0,1)(4.5,5]$
(ii) moving downward? $(2,4.5)$
(iii) at rest? $[1,2]$
(b) State the acceleration of the particle at the specified times. Include units. acceleration = derivative of velocity
(i) $t = 0.75$ slope = $\boxed{-2}$
(ii) $t = 4.2$ 5 (slope between $x=4$ and $x=5$)

7. (no calculator)
Given: $g(x) = f(x)\cdot\tan x + kx$, where k is a real number.
f is differentiable for all x; $f\left(\dfrac{\pi}{4}\right) = 4$; $f'\left(\dfrac{\pi}{4}\right) = -2$.
(a) For what values of x, if any, in the interval $0 < x < 2\pi$ will the derivative of g fail to exist? Justify your answer.
(b) If $g'\left(\dfrac{\pi}{4}\right) = 6$, find the value of k.

8. The table provided below shows the position of a particle, S, at several times, t, as the particle moves along a straight line, where t is measured in seconds and S is measured in meters.

t	2.0	2.7	3.2	3.8
$S(t)$	5.2	7.8	10.6	12.2

Which of the following best estimates the velocity of the particle at $t = 3$?
(a) $9.2\,\dfrac{\text{m}}{\text{s}}$ (b) $7.8\,\dfrac{\text{m}}{\text{s}}$ (c) $5.6\,\dfrac{\text{m}}{\text{s}}$
$\dfrac{10.6-7.8}{3.2-2.7} = \dfrac{2.8}{.5}$

9. (no calculator)
If $2y^3 - 3xy + x^2 = 4$, then $\dfrac{dy}{dx} =$
(A) $-\dfrac{2x}{6y^2-3}$ (B) $\dfrac{2x-3y}{3x-6y^2}$ (C) $\dfrac{2x-3}{6y^2}$
(D) $-\dfrac{2x}{6y^2-3x}$ (E) $\dfrac{3y-2x}{6y^2}$

$6y^2\dfrac{dy}{dx} - \left[3x\dfrac{dy}{dx} + 3y\right] + 2x = 0$
$6y^2\dfrac{dy}{dx} - 3x\dfrac{dy}{dx} - 3y + 2x = 0$
$\dfrac{dy}{dx}(6y^2-3x) = -2x+3y$
$\dfrac{dy}{dx} = \dfrac{-2x+3y}{6y^2-3x}$
$\dfrac{-(2x-3y)}{-(-6y^2+3x)}$

7. $g(x) = f(x)\tan x + kx$

$\cancel{g'(\tfrac{\pi}{4}) = 6}$ $\quad f'(\tfrac{\pi}{4}) = -2 \quad f(\tfrac{\pi}{4}) = 4$

$g'(\tfrac{\pi}{4}) = 6$

$\cancel{g'(x) = f(x)\sec^2 x}$

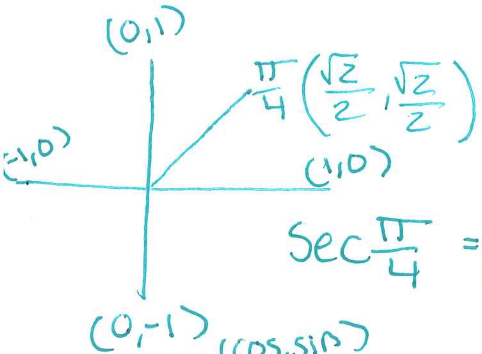
b. $g'(x) = f(x)\sec^2 x + \tan x\, f'(x) + k$

$6 = 4\sec^2\frac{\pi}{4} + \tan\frac{\pi}{4}(-2) + k$

$6 = 4(2) + 1(-2) + k$

$6 = -6 + k$

$\boxed{k = 12}$



$\sec\frac{\pi}{4} = \left(\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = 2$

a. $g'(x) = f(x)\sec^2 x + \tan x\, f'(x) + k$

@ $\frac{\pi}{2}, \frac{3\pi}{2}$ because $\tan\frac{\pi}{2}$ and $\tan\frac{3\pi}{2}$ are undefined.

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10. (no calculator)

The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. The radius and height of the cylinder are increasing at constant rates. The radius is expanding at $\frac{1}{3}\frac{\text{cm}}{\text{sec}}$ and the height is increasing at $\frac{1}{2}\frac{\text{cm}}{\text{sec}}$. At what rate, in cubic *cm* per second, is the volume of the cylinder increasing when its height is 9 cm and the radius is 4 cm?

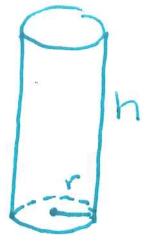
(A) 32π

(B) 6π

(C) $\frac{8\pi}{3}$

(D) $\frac{4\pi}{3}$

(E) $\frac{\pi}{18}$



$\frac{dr}{dt} = \frac{1}{3}$ Find $\frac{dV}{dt}$

$\frac{dh}{dt} = \frac{1}{2}$ when $h=9$, $r=4$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h\, 2\pi r \frac{dr}{dt}$$

$$\frac{dV}{dt} = \pi(16)\left(\tfrac{1}{2}\right) + (9)(2\pi)(4)\left(\tfrac{1}{3}\right)$$

$$\frac{dV}{dt} = 8\pi + 24\pi = 32\pi$$

1c. $24x(2x)(x^2-3)^2 + 8(x^2-3)^3$

$48x^2(x^2-3)^2 + 8(x^2-3)^3$

$\boxed{8(x^2-3)^2(6x^2+(x^2-3))}$

2a. $y=(4x-3)^{1/2}$

$y' = \frac{1}{2}(4x-3)^{-1/2}\cdot 4 = 2(4x-3)^{-1/2}$

$= \frac{2}{\sqrt{4x-3}} = \frac{2}{\sqrt{9}} = \boxed{\frac{2}{3}}$

2b. $g'(x) = \frac{xf'(x) - f(x)(1)}{x^2} = \frac{3\left(\frac{2}{9}\right) - 3}{3^2} = \frac{\frac{6}{9} - 3}{9}$

simplify

2c. Skip (But, normal = perpendicular slope = opp. rec.)

3a. $f(x) = \sin x$

$f'(0) = \cos(0) = 1$

3b. $f(x) = \sqrt[3]{x}$

$f'(0) = \frac{1}{3}x^{-2/3} = 0$

3c. 4

3d. $\frac{1}{5}$

4a. $\frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \frac{(5)(1)-(-3)(-2)}{5^2} = \frac{-1}{25}$

4b. $f'(g(x))g'(x) = f'(5)(-2) = (7)(-2) = -14$

4c. $\frac{1}{2}f(x)^{-1/2}\cdot f'(x) = \frac{1}{2\sqrt{f(x)}}\cdot f'(x) = \frac{1}{4}\cdot 7 = \frac{4}{7}$