Chapter 3 Review

Topic 1: Extrema

1. Find the **absolute extrema** for each of the following on $[-2, 5]$.
   a. $f(x) = 2x^3 - 6x$
   b. $f(x) = \frac{3x^2}{3-2x}$
2. Determine the intervals on which the function is increasing/decreasing and local (relative) extrema.
   a. $y = x^{3/4} - 3x$
   b. $y = (x-1)^2(x+3)$

Topic 2: Concavity and Points of Inflection

Determine the intervals on which the functions are concave up/down and any points of inflection.

1. $y = -x^3 + 6x^2 - 9x - 1$
2. $y = x + 2sinx, \ (-\pi, \pi)$

Topic 3: $y = h'(x)$

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

   (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   (b) On what intervals, if any, is the graph of h concave up? Justify your answer.

   (c) Write an equation for the line tangent to the graph of h at $x = 4$.

   (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

Topic 4: Speed/Velocity

3. An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

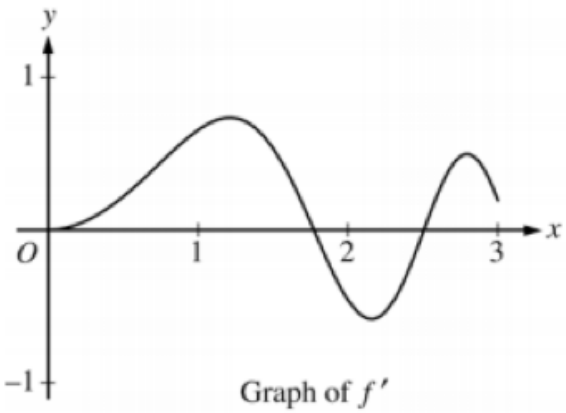
   (a) What is the acceleration of the object at time $t = 4$?

   (b) Consider the following two statements.

   Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
   Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

   Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

Topic 5: Graph of $f'(x)$



Graph of f'

2. Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin\left(x^2\right)$. The graph of $y = f'(x)$ is shown above.

   (a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

   (b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

   (c) Write an equation for the line tangent to the graph of f at $x = 2$.