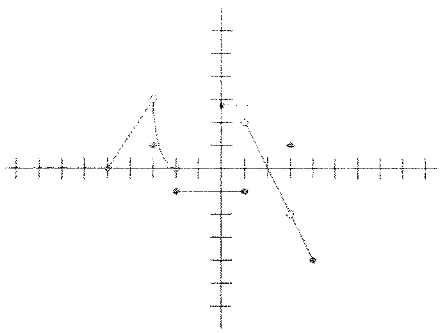
# Sem 1 Final Review – After School Review Session 1.26.17

ANSWER

## Limits – Graphically



1.) $\lim_{x \to 3} g(x)$

2.) $\lim_{x \to 0} g(x)$

3.) $\lim_{x \to -3} g(x)$

4.) $\lim_{x \to 1^+} g(x)$

5.) $\lim_{x \to 1^-} g(x)$

6.) $\lim_{x \to 1} g(x)$

7.) $\lim_{x \to -2^-} g(x)$

8.) $\lim_{x \to 4} g(x)$

9.) $\lim_{x \to 2} g(x)$

10.) $\lim_{x \to -2^+} g(x)$

## Limit – Evaluating Algebraically

7) $\displaystyle \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

8) $\displaystyle \lim_{x \to 5} \frac{x^2 - 5x}{x - 5}$

9) $\displaystyle \lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$

10) $\displaystyle \lim_{x \to -5} \frac{x^2 + 3x - 10}{x + 5}$

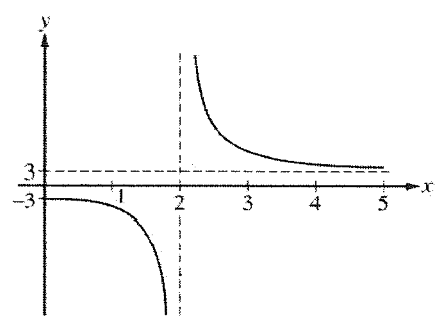
11) $\displaystyle \lim_{x \to 0} \frac{\frac{1}{-4 + x} + \frac{1}{4}}{x}$

12) $\displaystyle \lim_{x \to -3} \frac{x}{\frac{1}{3 + x} - \frac{1}{3}}$

13) $\displaystyle \lim_{x \to 5} \frac{x - 5}{\sqrt{x + 4} - 3}$

14) $\displaystyle \lim_{x \to 3} \frac{\sqrt{x + 6} - 3}{x - 3}$

## Limits – Horizontal and Vertical Asymptotes




10. The function $f$ is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of $f$. Which of the following could be the values of the constants $a$ and $b$?

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

1) $\lim_{x\to 3} g(x) = -2$

2) $\lim_{x\to 0} g(x) = -1$

3) $\lim_{x\to -3} g(x) = 3$

4) $\lim_{x\to 1^+} g(x) = 2$

5) $\lim_{x\to 1^-} g(x) = -1$

6) $\lim_{x\to 1} g(x)$ DNE

7) $\lim_{x\to -2^-} g(x) = 0$

8) $\lim_{x\to 4} g(x)$ DNE ($\lim_{x\to 4^+} g(x)$ DNE)

9) $\lim_{x\to 2} g(x) = 0$

10) $\lim_{x\to -2^+} g(x) = 0$

7) $\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x\to 1} x+1 = \boxed{2}$

9) $\lim_{x\to 2} -\frac{x^2-x-2}{x-2} = \lim_{x\to 2} -\frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} = \lim_{x\to 2} -(x+1) = \boxed{-3}$

11). $\lim_{x\to 0} \frac{\frac{1}{-4+x}+\frac{1}{4}}{x} = \lim_{x\to 0} \frac{\frac{4}{(-4+x)(4)} + \frac{-4+x}{(-4+x)(4)}}{x} = \lim_{x\to 0} \frac{\frac{4+-4+x}{(-4+x)(4)}}{x}$

$= \lim_{x\to 0} \frac{\frac{\cancel{x}}{(-4+x)(4)}}{\cancel{x}} = \lim_{x\to 0} \frac{1}{(-4+x)(4)} = \boxed{-\frac{1}{16}}$

13) $\lim_{x\to 5} \frac{x-5}{\sqrt{x+4}-3} = \lim_{x\to 5} \frac{(x-5)(\sqrt{x+4}+3)}{x+4-9} = \lim_{x\to 5} \frac{\cancel{(x-5)}(\sqrt{x+4}+3)}{\cancel{(x-5)}}$

$= \boxed{6}$

10) From the graph: V.A. at $x=2$  H.A at $y=3$

From the equation: V.A: $x^2+b=0 \to 2^2+b=0 \to \boxed{b=-4}$

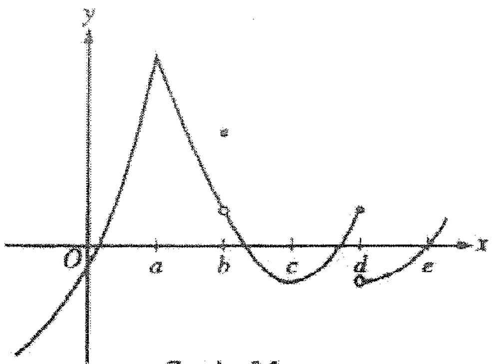
$\lim_{x\to\infty} \frac{ax^2+12}{x^2+b} = \lim_{x\to\infty} \frac{a\cancel{x^2}}{\cancel{x^2}} = \lim_{x\to\infty} a = a$

H.A. at $y = \boxed{a = 3}$

Continuity

21. Find the constant(s) $c$ for which the function $f(x)$ is continuous on $(-\infty,\infty)$.

$$f(x)=\begin{cases} c^2-x^2 & \text{if } x<2 \\ 2(x+c) & \text{if } x\geq 2 \end{cases}$$




Graph of $f$

4.

The graph of a function $f$ is shown above. At which value of $x$ is $f$ continuous, but not differentiable?

a. $a$
b. $b$
c. $c$
d. $d$
e. $e$

5. $\lim\limits_{x\to\infty} \dfrac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}=$

a. 4
b. 1
c. $\frac{1}{4}$
d. 0
e. -1

6. Let $f$ be the function defined by

$$f(x)=\begin{cases} \sqrt{x+1} & \text{for } 0\leq x\leq 3 \\ 5-x & \text{for } 3<x\leq 5. \end{cases}$$

(a) Is $f$ continuous at $x=3$ ? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0\leq x\leq 5$.

(c) Suppose the function $g$ is defined by

$$g(x)=\begin{cases} k\sqrt{x+1} & \text{for } 0\leq x\leq 3 \\ mx+2 & \text{for } 3<x\leq 5, \end{cases}$$

where $k$ and $m$ are constants. If $g$ is differentiable at $x=3$, what are the values of $k$ and $m$ ?

21.

f(x) is continuous means

at $x=2$, $c^2 - x^2 = 2(x+c)$

$$c^2 - 4 = 2(2+c)$$

$$c^2 - 4 = 4 + 2c$$

$$c^2 - 2c - 8 = 0$$

$$(c+2)(c-4) = 0$$

$$\boxed{c = -2 \quad \text{OR} \quad 4}$$

4.

At a, f(x) is continuous but NOT differentiable $\boxed{A}$

(sharp turn)

5. $\lim\limits_{x\to\infty} \dfrac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^3}}{4\cancel{x^3}} = \lim\limits_{x\to\infty} \dfrac{1}{4} = \boxed{\dfrac{1}{4}}$ $\boxed{C}$

6. a) At $x=3$, $\sqrt{x+1} = \sqrt{3+1} = \sqrt{4} = 2$
$5-x = 5-3 = 2$ > equal. Yes, f is continuous at $x=3$.

b). Skip this Part!!

c). Differentiable → continuous <u>and</u> same slope.

$$\begin{cases} k\sqrt{x+1} = mx+2 \\ \frac{1}{2}k(x+1)^{-\frac{1}{2}} = m \end{cases}$$

continuous, two Pieces have same value.

Same slope, derivatives of the 2 Pieces are same.

$$\begin{cases} k\sqrt{3+1} = 3m+2 \\ \frac{1}{2}k(3+1)^{-1/2} = m \end{cases}$$

Plug in 3 for x.

$$\begin{cases} 2k = 3m+2 \\ \frac{1}{4}k = m \end{cases} \to 2k = 3(\tfrac{1}{4}k)+2 \to \tfrac{5}{4}k = 2 \to \boxed{\begin{array}{l} k = \frac{8}{5} \\ m = \frac{1}{4}k = \frac{2}{5} \end{array}}$$

Chapter 2 – Derivatives

Rules of Derivatives

## Examples

1. If $y = \left(x^3 + 1\right)^2$, then $\frac{dy}{dx} =$

   a. $\left(3x^2\right)^2$
   b. $2\left(x^3 + 1\right)$
   c. $2\left(3x^2 + 1\right)$
   d. $3x^2\left(x^3 + 1\right)$
   e. $6x^2\left(x^3 + 1\right)$

6. If $f(x) = x\sqrt{2x - 3}$, then $f'(x) =$

   a. $\frac{3x - 3}{\sqrt{2x - 3}}$
   b. $\frac{x}{\sqrt{2x - 3}}$
   c. $\frac{1}{\sqrt{2x - 3}}$
   d. $\frac{-x + 3}{\sqrt{2x - 3}}$
   e. $\frac{5x - 6}{2\sqrt{2x - 3}}$

1. $\frac{dy}{dx} = 2(x^3+1)\underbrace{(3x^2)}_{\uparrow}$ chain Rule $\boxed{E}$

6. $f'(x) = x\left(\frac{1}{2}(2x-3)^{-1/2}\right) + \sqrt{2x-3}$ Product Rule.

$= \frac{x}{2\sqrt{2x-3}} + \frac{2x-3}{\sqrt{2x-3}}$ Equivalent. Rewrite to match answer choices.

$= \frac{x+2(2x-3)}{2\sqrt{2x-3}} = \frac{5x-3}{2\sqrt{2x-3}}$ $\boxed{E}$

12.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| $-1$ | 6 | 5 | 3 | $-2$ |
| 1 | 3 | $-3$ | $-1$ | 2 |
| 3 | 1 | $-2$ | 2 | 3 |

The table above gives values of $f$, $f'$, $g$, and $g'$ at selected values of $x$. If $h(x) = f(g(x))$, then $h'(1) =$

a. 5
b. 6
c. 9
d. 10
e. 12

## Implicit Differentiation

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point $P$ with $x$-coordinate 3 at which the line tangent to the curve at $P$ is horizontal. Find the $y$-coordinate of $P$.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point $P$ found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point $P$? Justify your answer.

## Related Rates

5.) **A conical tank (with vertex down) is 12 feet across and 10 feet deep. If water is flowing into the tank at a rate of 5 cubic feet per minute, find the rate of change of the depth of the water when the water is 6 feet deep?**

6.) **A rocket, rising vertically, is tracked by a radar station that is on the ground 3000 feet from the launching pad. At what rate is the angle of elevation changing when the rocket is 4000 feet up and rising vertically at 5000 feet per second?**

12. $h'(x) = f'(g(x))\, \underbrace{g'(x)}_{\uparrow \text{ chain Rule}}$

$h'(1) = f'(g(1))\, g'(1)$

$= f'(-1)\,(2) = (5)(2) = 10$ $\boxed{D}$

4.

a) $2x + 8y \frac{dy}{dx} = 0 + \underbrace{3x \frac{dy}{dx} + 3y}_{\text{Product Rule}}$

$8y \frac{dy}{dx} - 3x \frac{dy}{dx} = 3y - 2x$

$\frac{dy}{dx}(8y - 3x) = 3y - 2x \rightarrow \frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$

b) horizontal tangent. $\rightarrow \frac{dy}{dx} = 0$

$\frac{3y-2x}{8y-3x} = 0 \rightarrow 3y - 2x = 0$

$x = 3 \rightarrow 3y - 2x = 3y - (2)(3) = 0 \rightarrow \boxed{y = 2}$

c). $\frac{d^2y}{dy^2} = \frac{(8y-3x)(3\frac{dy}{dx} - 2) - (3y-2x)(8\frac{dy}{dx} - 3)}{(8y-3x)}$ ← Quotient Rule.

$= \frac{[(8)(2)-(3)(3)][(3)(0)-2] - [(3)(2)-(2)(3)][(8)(0)-3]}{(8)(2)-(3)(3)}$ ← plug in values. $x=3$, $y=2$, $\frac{dy}{dx} = 0$

$= \frac{(16-9)(-2) - (0)(-3)}{16-9} = \frac{-14}{7} = \boxed{-2}$

2nd derivative $= -2 < 0 \rightarrow f$ is concave down.

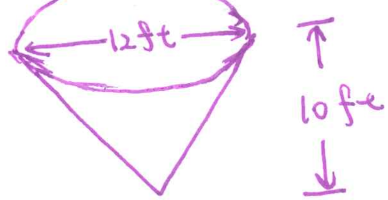
1st derivative $= 0 \rightarrow f$ has slope $= 0$

$\rightarrow$  local min at P

by the second derivative test.

Related Rates

5.




Tank.

$h_T = 10$ ft

At top, $d_T = 12$ ft $(r_T = 6$ ft$)$

Water.

$h = 6$ ft (changing)

$\frac{dV}{dt} = 5$ ft$^3$/min. (changing)

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{3}{5}h\right)^2 h$

$V = \frac{9}{75}\pi h^3$

$\frac{dV}{dt} = \frac{9}{75}\pi\left(3h^2 \frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{9}{25}\pi h^2 \frac{dh}{dt}$

$5 = \frac{9}{25}\pi (6^2) \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} \approx 0.113 \text{ ft/min}}$

Also need r water to plug in

But we cannot solve for r water using $V = \frac{1}{3}\pi r^2 h$ b/c we don't know h either!!

But, from the tank, we know

$\frac{r_T}{h_T} = \frac{6}{10} = \frac{3}{5}$

so $r = \frac{6}{10}h = \frac{3}{5}h$

This ratio is also true for water.

For water, r, h both are changing, but the ratio stays the same.

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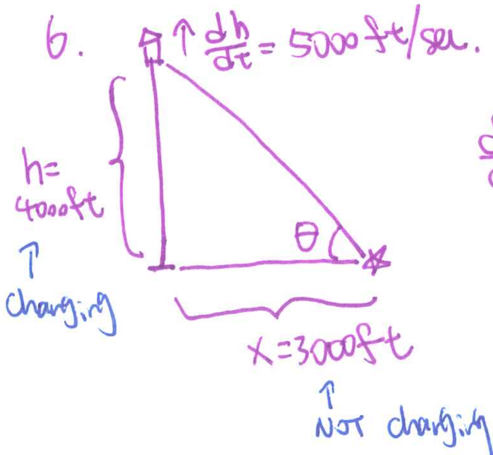
6.




$\frac{d\theta}{dt} = ?$

diag hypotenuse $= 5000$ ft.

$\sec\theta = \frac{5000}{3000} = \frac{5}{3}$

$\tan\theta = \frac{h}{x} = \frac{h}{3000}$ derivative.

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{3000}\frac{dh}{dt}$

$\left(\frac{5}{3}\right)^2 \frac{d\theta}{dt} = \frac{1}{3000} 5000$

$\boxed{\frac{d\theta}{dt} = \frac{3}{5} \text{ rad/sec.}}$

## First derivative Analysis

## Second Derivative Analysis

## Examples

**701.** $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x - 1$

**702.** $g(x) = x^3 - 5x^2 - 8x$

**703.** $h(x) = x + \dfrac{4}{x}$

**704.** $p(x) = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$

**705.** $h(x) = (2 - x)^2(x + 3)^3$

**706.** $m(x) = 3x\sqrt{5 - x}$

**707.** $f(x) = x^{2/3}(x - 5)^{-1/3}$

**708.** $h(x) = \frac{1}{7}x^{7/3} - x^{4/3}$

701.

$f'(x) = x^2 + 5x + 6$

702.

$g'(x) = 3x^2 - 10x - 8$

703 $h(x) = x + \frac{4}{x} = x + 4x^{-1}$

$h'(x) = 1 - 4x^{-2} = 1 - \frac{4}{x^2}$

704 $p(x) = x^{1/3} + x^{-1/3}$

$p'(x) = \frac{1}{3}x^{-2/3} - \frac{1}{3}x^{-4/3}$ OR $\frac{1}{3\sqrt[3]{x^2}} - \frac{1}{3\sqrt[3]{x^4}}$

705 $h'(x) = 2(2-x)\underbrace{(-1)}_{\uparrow \text{chain Rule}}(x+3)^3 + (2-x)^2(3(x+3)^2)$ Product Rule

$= -2(2-x)(x+3)^3 + 3(2-x)^2(x+3)^2 = \underbrace{(2-x)(x+3)^2}_{\uparrow}\left[-2(x+3) + 3(2-x)\right]$

$= (2-x)(x+3)^2(-5x)$

Factored these out from both terms.

706 $m(x) = 3x(5-x)^{1/2}$

$m'(x) = 3x\left(\frac{1}{2}(5-x)^{-1/2}\right) + 3(5-x)^{1/2}$

$= \frac{3}{2}x(5-x)^{-1/2} + 3(5-x)^{1/2} \cdot \frac{\sqrt{5-x}}{\sqrt{5-x}}$ ← to help simplify...

$= \frac{3x}{2\sqrt{5-x}} + \frac{3(5-x)}{\sqrt{5-x}} \cdot \frac{2}{2} = \frac{3x + 6(5-x)}{2\sqrt{5-x}}$

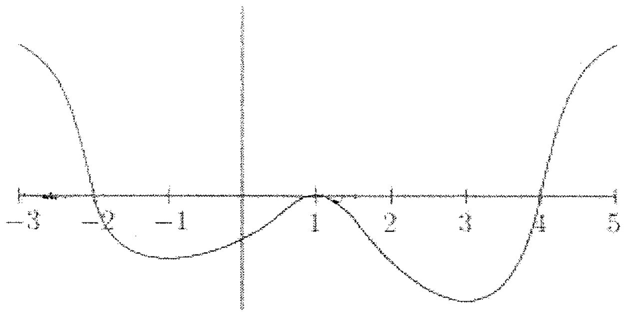
$= \frac{30-3x}{2\sqrt{5-x}}$ ← more simplified.

708

$h'(x) = \frac{1}{3}x^{4/3} - \frac{4}{3}x^{1/3}$

Graphs of f'

813 (1996AB). The figure below shows the graph of $f'$, the derivative of a function $f$. The domain of $f$ is the set of all real numbers $x$ such that $-3 < x < 5$.




a) For what values of $x$ does $f$ have a relative maximum? Why?

b) For what values of $x$ does $f$ have a relative minimum? Why?

c) On what intervals is the graph of $f$ concave upward? Use $f'$ to justify your answer.

d) Suppose that $f(1) = 0$. Draw a sketch of $f$ that shows the general shape of the graph on the open interval $0 < x < 2$.

## Motion Applications

1. A particle moves along a line so that at any time $t$ its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

   a) Find the velocity at time $t$.

   b) Find the acceleration at time $t$.

   c) What are all values of $t$, for $0 \le t \le 3$, for which the particle is at rest?

   d) What is the maximum velocity?

813.

a) f has a relative max when f' changes from ⊕ to ⊖

at $x = -2$

b) f has a relative min when f' changes from ⊖ to ⊕

at $x = 4$

Motion

1. a). $V(t) = 2\pi + -\sin(2\pi t)\,(2\pi)$ ← chain Rule.

$= 2\pi - 2\pi\sin(2\pi t)$

b) $a(t) = 0 - 2\pi\cos(2\pi t)(2\pi)$ ← chain Rule again!

$= -4\pi^2\cos(2\pi t)$

c) At rest → $V(t) = 0$

$2\pi - 2\pi\sin(2\pi t) = 0$

$\cancel{2}\pi\sin(2\pi t) = \cancel{2}\pi$

$\sin(2\pi t) = 1$

$2\pi t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \ldots$

$\boxed{t = \frac{1}{4}, \frac{5}{4}, \frac{9}{4},}\ \cancel{\frac{13}{4}}, \ldots$

↑ NOT on the interval $0 \le t \le 3$

d). V(t) at max.

$\Rightarrow V'(t) = a(t) = 0$

and $V'(t) = a(t)$ goes from ⊕ to ⊖

$-4\pi^2\cos(2\pi t) = 0$

$\cos(2\pi t) = 0$

$2\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$t = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$

| t | | 1/4 | | 3/4 | | 5/4 | | 7/4 | ... |
|---|---|---|---|---|---|---|---|---|---|
| V'(t) | − | | + | | − | | + | | − ... |
| | | | | max | | | | max | |

$V(t) = 2\pi - 2\pi\sin(2\pi(\frac{3}{4})) = \boxed{4\pi}$ ← you will get $4\pi$ if you plug in $\frac{7}{4}$, $\frac{11}{4}$, ......

Chapter 4

## Indefinite Integrals

11) $\int \left(-\frac{4}{x^3} - \frac{8}{x^5}\right) dx$

12) $\int \left(\frac{15}{x^4} + \frac{8}{x^5}\right) dx$

13) $\int -\frac{14x^{\frac{5}{2}}}{2}\, dx$

14) $\int -\frac{35x^{\frac{2}{5}}}{5}\, dx$

15) $\int -\frac{5\sqrt[3]{x^2}}{3}\, dx$

16) $\int -\frac{5\sqrt[4]{x}}{2}\, dx$

## Riemann Sums

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function $R$ of time $t$. The table above shows the rate as measured every 3 hours for a 24-hour period.
   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\, dt$. Using correct units, explain the meaning of your answer in terms of water flow.
   (b) Is there some time $t$, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

| $t$ (hours) | $R(t)$ (gallons per hour) |
|---|---|
| 0 | 9.6 |
| 3 | 10.4 |
| 6 | 10.8 |
| 9 | 11.2 |
| 12 | 11.4 |
| 15 | 11.3 |
| 18 | 10.7 |
| 21 | 10.2 |
| 24 | 9.6 |

11. $\int \left(-\frac{4}{x^3} - \frac{8}{x^5}\right) dx$

$= \int -4x^{-3} - 8x^{-5}\, dx$

$= 2x^{-2} + 2x^{-4} + C$

12. $\int 15x^{-4} + 8x^{-5}\, dx$

$= -5x^{-3} - 2x^{-4} + C$

13. $\int -\frac{14x^{5/2}}{2}\, dx$

$= \int -\frac{14}{2} x^{5/2} dx = -7x^{5/2} dx$

$= -2x^{7/2} + C$

14. $\int -7x^{2/5}\, dx$

$= -5x^{7/5} + C$

15. $\int -\frac{5\sqrt[3]{x^2}}{3}\, dx$

$= \int -\frac{5}{3} x^{2/3}\, dx$

$= -x^{5/3} + C$

16. $\int -\frac{5}{2} x^{1/4}\, dx$

$= -2x^{5/4} + C$.

Riemann Sums

3.

a)

| | $t$ | $R(t)$ |
|---|---|---|
| | 0 | $\boxed{9.6}$ |
| $\Delta x=6$ | 3 | 10.4 |
| | 6 | 10.8 |
| $\Delta x=6$ | 9 | 11.2 |
| | 12 | 11.4 |
| $\Delta x=6$ | 15 | 11.3 |
| | 18 | 10.7 |
| $\Delta x=6$ | 21 | 10.2 |
| | 24 | $\boxed{9.6}$ |

midpoints.

$\int_0^{24} R(t)dt \approx (6)(10.4) + (6)(11.2) + (6)(11.3) + (6)(10.2) = 258.6$ Gallons.

b). Average rate of change of R on $(0, 24)$ $= \frac{\Delta R}{\Delta t} = \frac{9.6-9.6}{24-0}$

$= 0$

By MVT, yes, there IS some time $t$, $0<t<24$ such that $R'(t)=0$

c) Skip this part !!

Definite Integrals

3) $\int_{1}^{3} (2x^2 - 12x + 13)\, dx$

4) $\int_{0}^{3} (-x^3 + 3x^2 - 2)\, dx$

5) $\int_{-1}^{0} (x^5 - 4x^3 + 4x + 4)\, dx$

6) $\int_{-3}^{0} 4x^{\frac{1}{3}}\, dx$

Total Distance Traveled

1. The velocity of a particle moving on a line at time $t$ is $v = 5t^{\frac{2}{3}} + 6t$. How many meters did the particle travel from $t = 1$ to $t = 8$?

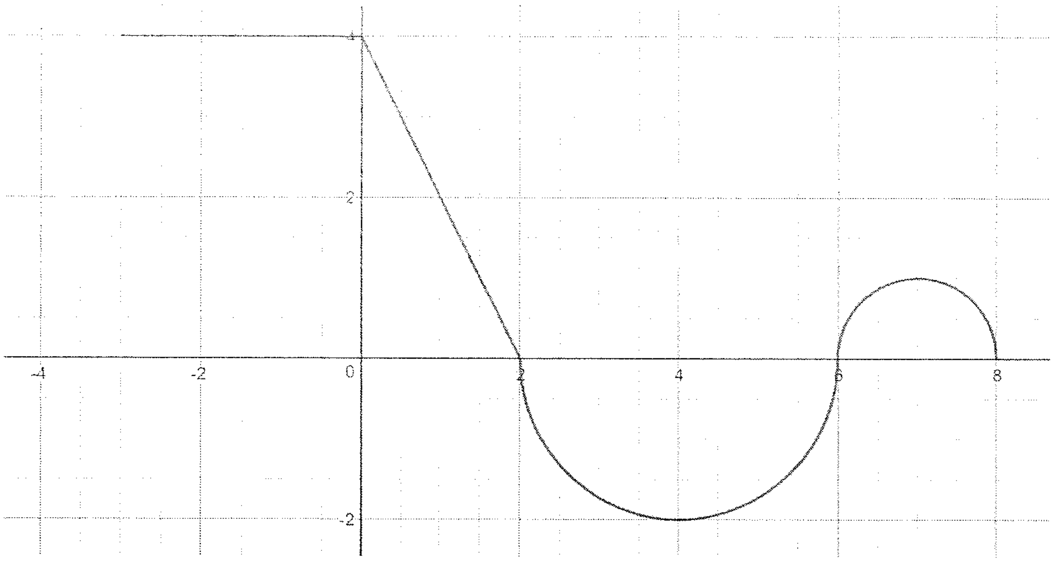
(A) $-\frac{10}{3}$ (B) 224 (C) 279

(D) 282 (E) 533

Let $v(t) = \frac{1}{\pi} + \sin 3t$ represent the velocity of an object moving on a line. At $t = \frac{\pi}{3}$, the position is 4.

Find the total distance traveled on [0, 5].

Definite Integrals – Evaluating Geometrically – Use the graph of $y = f'(x)$ below which consists of line segments and semicircles to find the following definite integrals.




| | | |
|---|---|---|
| $\int_0^6 f'(x)dx$ | $\int_{-2}^4 f'(x)dx$ | $\int_8^6 f'(x)dx$ |
| $\int_2^2 f'(x)dx$ | $\int_{-4}^8 2f'(x) + 3dx$ | $\int_4^1 (f'(x) - x)dx$ |

1. On what interals is $f$ increasing? Decreasing? Justify.

2. What are the local extrema of $f$? Justify.

3. On what interval is $f$ concave up/down? Justify.

4. What are the points of inflection of $f$? Justify.

Evaluate – Geometrically by graphing

$\int_1^3 2x + 1\, dx$ $\int_0^4 x - 1\, dx$ $\int_0^5 \sqrt{25 - x^2}dx$

a) $\int_0^6 f'(x)\,dx = \underbrace{\tfrac{1}{2}(2)(4)}_{\triangle \text{ above x-axis}} - \underbrace{\tfrac{1}{2}\pi 2^2}_{\text{semi }\bigcirc\text{ below x-axis}} = 4-2\pi$

b) $\int_{-2}^{4} f'(x)\,dx = \underbrace{(2)(4)}_{\square \text{ above x-axis}} + \underbrace{\tfrac{1}{2}(2)(4)}_{\triangle \text{ above x-axis}} - \underbrace{\tfrac{1}{4}\pi 2^2}_{\frac{1}{4}\bigcirc \text{ below x-axis}} = 12-\pi$

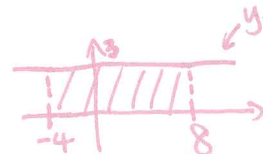
c) $\int_8^6 f'(x)\,dx = -\int_6^8 f'(x)\,dx$

$= -\left[\tfrac{1}{2}\pi 1^2\right] = -\tfrac{1}{2}\pi$

d) $\int_2^2 f'(x)\,dx = 0$

e) $\int_{-4}^{8} 2f'(x)+3\,dx = \int_{-4}^{8} 2f'(x)\,dx + \int_{-4}^{8} 3\,dx$

$= 2\underbrace{\int_{-4}^{8} f'(x)\,dx} + \underbrace{\int_{-4}^{8} 3\,dx}$

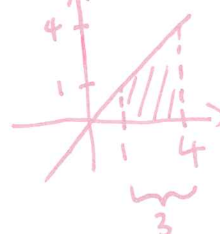



$= 2\left[16+4-2\pi+\tfrac{1}{2}\pi\right] + (12)(3)$

$= 76 - \tfrac{3}{2}\pi$

f) $\int_4^1 (f'(x)-x)\,dx = -\int_1^4 (f'(x)-x)\,dx = -\left[\int_1^4 f'(x)\,dx - \int_1^4 x\,dx\right]$

$= -\underbrace{\int_1^4 f'(x)\,dx} + \underbrace{\int_1^4 x\,dx}$




$= -(1-\pi) + (1+4)(3)(\tfrac{1}{2})$

$= \frac{13}{2} + \pi$

Remember we're looking at the graph of $f'(x)$ !!!

1. $f$ inc on $(-4,2) \cup (6,8)$ $\quad f'$ $+$

$f$ dec on $(2,6)$ $\quad f'$ $-$

2. $f$ local max at $x=2$ $\quad f'$ goes $+ \rightarrow -$

$f$ local min at $x=6$ $\quad f'$ goes $- \rightarrow +$

3. $f$ concave up on $(4,7)$ $\quad f'$ inc

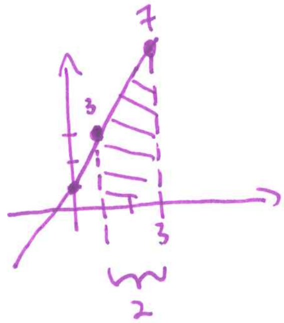
$f$ concave down $(0,4) \cup (7,8)$ $\quad f'$ dec.

4. $x$ coord. of pts of inflections of $f$: $x=4$ , $x=7$

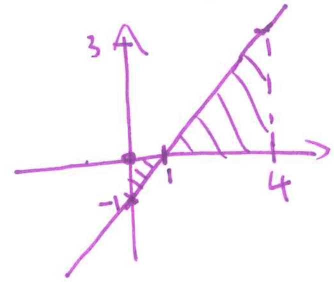
that's when concavity of $f$ changes

($f'$ goes from inc to dec)

a) $\int_1^3 2x+1\,dx = \frac{1}{2}(3+7)(2) = 10$




b) $\int_0^4 x-1\,dx = -\frac{1}{2} + 3 = \frac{5}{2}$




c). $\int_0^5 \sqrt{25-x^2}\,dx = \frac{1}{4}(\pi 5^2) = \frac{25}{4}\pi$