

9.1 Circles: Notes and Practice

Definition of a Circle

A **circle** is the set of all points (x, y) in a plane that are equidistant from a fixed point (h, k) , called the **center** of the circle. (See Figure 9.3.) The distance r between the center and any point (x, y) on the circle is the **radius**.

Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

$$(x - h)^2 + (y - k)^2 = r^2.$$

The point (h, k) is the center of the circle, and the positive number r is the radius of the circle. The standard form of the equation of a circle whose center is the origin, $(h, k) = (0, 0)$, is

$$x^2 + y^2 = r^2.$$

<p>Example by me:</p>	<p>You try something similar:</p>
<p>Example 1: The point $(1, 4)$ is on a circle whose center is at $(-2, -3)$. Write the standard form of the equation of the circle.</p> <p><i>We can find the radius using the distance formula:</i></p> $\sqrt{(1-(-2))^2 + (4-(-3))^2} = \sqrt{9+49}$ $= \sqrt{58} = r$ $(x+2)^2 + (y+3)^2 = 58$	<p>You try! The point $(0, 1)$ is on a circle whose center is at $(-3, -2)$. Write the standard form of the equation of a circle.</p>
<p>Example 2: Find the x- and y- intercepts of the graph of the circle given by the equation $(x - 4)^2 + (y - 2)^2 = 16$</p> <p><i>x-int = when y=0</i> $(x-4)^2 + (0-2)^2 = 16$ $(x-4)^2 + 4 = 16$ $(x-4)^2 = 12$ $x-4 = \pm 2\sqrt{3}$ $x = (\pm 2\sqrt{3} + 4, 0)$</p> <p><i>y-int = when x=0</i> $(-4)^2 + (y-2)^2 = 16$ $16 + (y-2)^2 = 16$ $(y-2)^2 = 0$ $y-2 = 0$ $y = 2$, so $(0, 2)$</p>	<p>You try! Find the x- and y- intercepts of the graph of the circle given by the equation $(x + 3)^2 + y^2 = 16$</p>
<p>Example 3: Determine the center and the radius of a circle with equation $x^2 + y^2 = 49$ Center = $(0, 0)$ Radius = $\sqrt{49} = 7$</p>	<p>You try! Determine the center and the radius of a circle with equation $(x - 1)^2 + (y - 2)^2 = 16$</p>

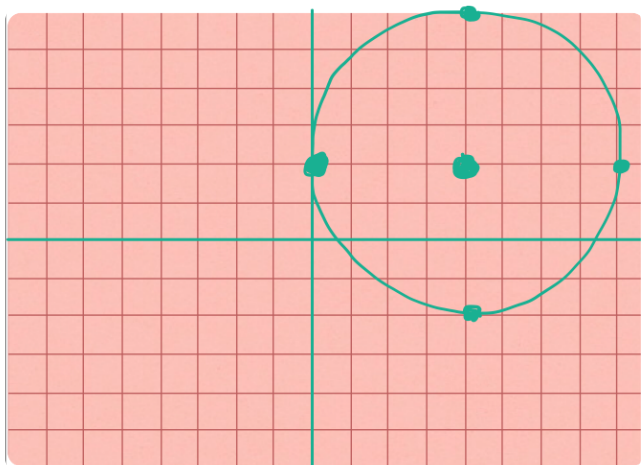
Example 4: Write the standard form of the equation of a circle given the center at $(-3, -1)$ and radius $4\sqrt{2}$.

$$(x+3)^2 + (y+1)^2 = (4\sqrt{2})^2$$

$$(x+3)^2 + (y+1)^2 = 32$$

You try! Write the standard form of the equation of a circle given the center at $(1, 2)$ and radius of 3.

Example 5: Graph the circle in Example 2.



You try! Graph the circle in Example 3.

Example 6: Identify the center and radius of a circle given by $4x^2 + 4y^2 + 12x - 24y + 41$. Then, graph the circle.

Complete the square for x and y . But first, regroup x and y together.

$$4x^2 + 12x + 4y^2 - 24y = -41$$

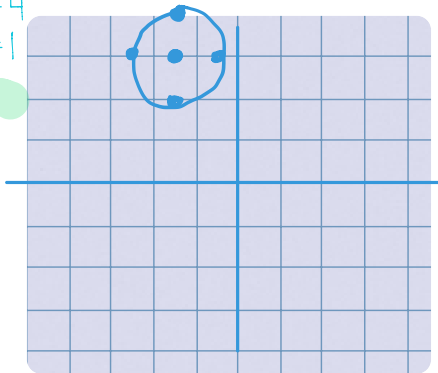
$$4(x^2 + 3x + \frac{9}{4}) + 4(y^2 - 6y + 9) = -41 + 9 + 36$$

$$4(x + \frac{3}{2})^2 + 4(y - 3)^2 = 4$$

$$(x + \frac{3}{2})^2 + (y - 3)^2 = 1$$

Center @ $(-\frac{3}{2}, 3)$

Radius = 1



You try! Identify the center and radius of a circle given by $x^2 + y^2 + 10y + 9$. Then, graph the circle.

Hint: only complete the square with y 's this time.