9.1 Circles: Notes and Practice

Definition of a Circle

A **circle** is the set of all points (x, y) in a plane that are equidistant from a fixed point (h, k), called the **center** of the circle. (See Figure 9.3.) The distance r between the center and any point (x, y) on the circle is the **radius.**

Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

$$(x - h)^2 + (y - k)^2 = r^2.$$

The point (h, k) is the center of the circle, and the positive number r is the radius of the circle. The standard form of the equation of a circle whose center is the origin, $(h, k) = (0, 0)$, is

$$x^2 + y^2 = r^2.$$

Example by me:	You try something similar:
Example 1: The point $(1,\ 4)$ is on a circle whose center is at $(-2,\ -3)$. Write the standard form of the equation of the circle. We can find the radius using the distance formula: $\sqrt{(1--2)^2+(4--3)^2} = \sqrt{9+49}$ $= \sqrt{58} = r$ $\boxed{(x+2)^2+(y+3)^2=58}$ r^2	**You try!** The point $(0,\ 1)$ is on a circle whose center is at $(-3,\ -2)$. Write the standard form of the equation of a circle. $(x-h)^2+(y-k)^2=r^2$ $(x+3)^2+(y+2)^2=18$ $\sqrt{(0--3)^2+(1--2)^2}=\sqrt{18}$
Example 2: Find the x- and y- intercepts of the graph of the circle given by the equation $(x-4)^2+(y-2)^2=16$ x-int = when y=0 $(x-4)^2+(0-2)^2=16$ $(x-4)^2+4=16$ $(x-4)^2=12$ $x-4=\pm 2\sqrt{3}$ $x=(\pm 2\sqrt{3}+4, 0)$ y-int = when x=0 $(-4)^2+(y-2)^2=16$ $16+(y-2)^2=16$ $(y-2)^2=0$ $y-2=0$ $y=2$, so $(0,2)$	**You try!** Find the x- and y- intercepts of the graph of the circle given by the equation $(x+3)^2+y^2=16$ x-int $(x+3)^2=16$ $x+3=\pm 4$ $x=\pm 4-3$ $(1,0)$ $(-7,0)$ or $(\pm 4-3, 0)$ y-int $9+y^2=16$ $y^2=7$ $y=\pm\sqrt{7}$ $(0,\pm\sqrt{7})$
Example 3: Determine the center and the radius of a circle with equation $x^2+y^2=49$ Center=(0,0) Radius $=\sqrt{49}=7$	**You try!** Determine the center and the radius of a circle with equation $(x-1)^2+(y-2)^2=16$ Center = (1, 2) r = 4

Example 4: Write the standard form of the equation of a circle given the center at $(-3,\ -1)$ and radius $4\sqrt{2}$.

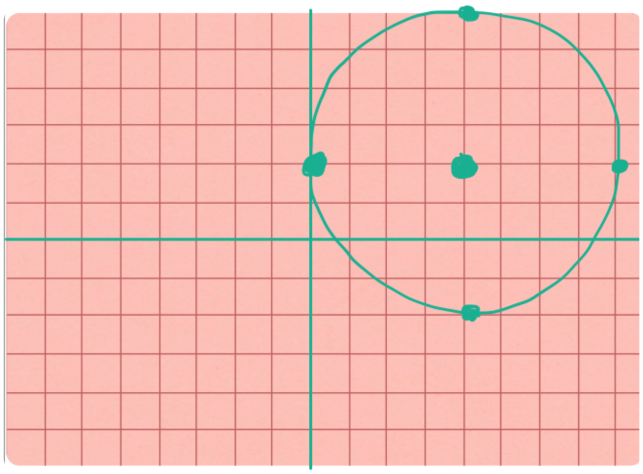
$(x+3)^2+(y+1)^2=(4\sqrt{2})^2$

$\boxed{(x+3)^2+(y+1)^2=32}$

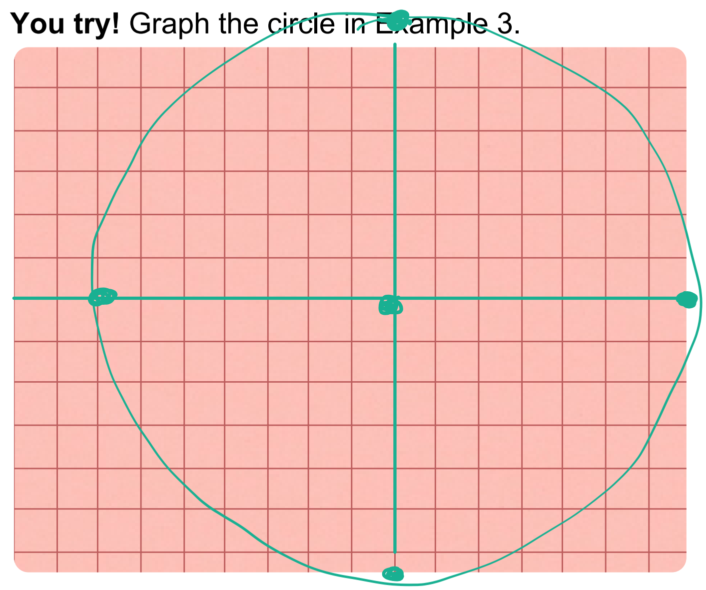
You try! Write the standard form of the equation of a circle given the center at $(1,\ 2)$ and radius of 3.

$(x-1)^2+(y-2)^2=9$

Example 5: Graph the circle in Example 2.



You try! Graph the circle in Example 3.



Example 6: Identify the center and radius of a circle given by $4x^2+4y^2+12x-24y+41=0$. Then, graph the circle.

complete the square for x and y. But first, regroup x and y together.

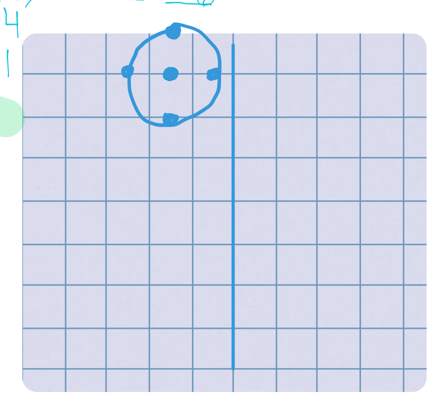
$4x^2+12x+4y^2-24y=-41$

$4(x^2+3x+\underline{9/4})+4(y^2-6y+\underline{9})=-41+\underline{9}+\underline{36}$

$4(x+3/2)^2+4(y-3)^2=4$

$(x+3/2)^2+(y-3)^2=1$

Center @ $(-3/2, 3)$
radius $=1$



You try! Identify the center and radius of a circle given by $x^2+y^2+10y+9=0$. Then, graph the circle.

Hint: only complete the square with y's this time

$(x-h)^2+(y-k)^2=r^2$

$x^2+y^2+10y=-9$

$y^2+10y+\underline{25}=-9+\underline{25}$

$x^2+(y+5)^2=16$

Center @ $(0,-5)$ $r=4$