8.1 Sequences and Series

Homework: None

Warm-Up: # 54, 56, 58 ↓

a_{k-1} a_k a_{k+1}

A Recursive Sequence **In Exercises 53–58, write the first five terms of the sequence defined recursively.**

53. $a_1 = 28, \quad a_{k+1} = a_k - 4$

54. $a_1 = 15, \quad a_{k+1} = a_k + 3$

55. $a_1 = 3, \quad a_{k+1} = 2(a_k - 1)$

56. $a_1 = 32, \quad a_{k+1} = \frac{1}{2}a_k$

57. $a_0 = 1, a_1 = 3, \quad a_k = a_{k-2} + a_{k-1}$

58. $a_0 = -1, a_1 = 1, \quad a_k = a_{k-2} + a_{k-1}$

54. $15+3=18=a_2$
$a_3 = 18+3=21$
$a_4 = 21+3=24$
$a_5 = 24+3=27$

56. $a_2 = \frac{1}{2}(32) = 16$
$a_3 = \frac{1}{2}(16) = 8$
$a_4 = 4$
$a_5 = 2$

58. $a_2 = a_{2-2} + a_{2-1} = 0$
$a_3 = 1$
$a_4 = 1$
$a_5 = 2$

Simplifying Factorial Expressions:

Example 1: $\frac{8!}{2!\cdot 6!}$

$\frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$

$\frac{56}{2} = 28$

Example 2: $\frac{n!}{(n-1)!}$

$\frac{3!}{2!} = \frac{3\cdot 2\cdot 1}{2\cdot 1}$

$= 3$

$= n$

Sigma

Definition of Summation Notation

The sum of the first n terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$$

where i is called the **index of summation**, n is the **upper limit of summation**, and 1 is the **lower limit of summation.**

"Sum from $i=1$ to n of ..."

Example 3: $\sum_{i=1}^{5} 3i$

"the sum from i=1 to 5 of 3i"

$3(1)+3(2)+3(3)+3(4)+3(5)$

$3+6+9+12+15=45$

Example 4: $\sum_{k=3}^{6} (1+k^2)$

$(1+3^2)+(1+4^2)+(1+5^2)+(1+6^2)$

$10+17+26+37=90$

Example 5: $\sum_{n=0}^{8} \frac{1}{n!}$

$\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}$

$0!=1$

Properties of Sums:

Properties of Sums (See the proofs on page 632.)

1. $\sum_{i=1}^{n} c = cn$, c is a constant.
2. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$, c is a constant.
3. $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$
4. $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

1. $\sum_{i=1}^{4} 5 = 20$ 2. $\sum_{i=1}^{4} 5i = 5\sum_{i=1}^{4} i$

Homework

Practice:

87. $\sum_{i=1}^{5}(2i+1)$

89. $\sum_{k=1}^{4} 10$

91. $\sum_{i=0}^{4} i^2$

93. $\sum_{k=0}^{3} \frac{1}{k^2+1}$

95. $\sum_{i=1}^{4}[(i-1)^2+(i+1)^3]$

97. $\sum_{i=1}^{4} 2^i$