Discovering the Double-Angle Formulas

Part 1: What is a double-angle?

Part 2: Sine of a double-angle → Sin(💃 + 💃)

a. Recall from the last section, the sine of the sum of two different angles: $sin(\alpha+\beta) =$ ________________

b. Rewrite the left side of the equation in step a so that we are adding two of the same angles by replacing β with α : ______________ $= sin2\alpha$

c. Since we replaced β in the left side, we need to do the same on the right. We get:

____

↑ Be sure to simplify (there are terms you can combine)! ↑

d. Put the results together to get our first double-angle formula:

$sin2\alpha =$ $2sin\alpha cos\alpha$

$sin\alpha cos\alpha + sin\alpha cos\alpha$

Part 3: Cosine of a double-angle→ Cos(🍕 + 🍕)

a. Recall from the last section, the cosine of the sum of two angles: $cos(\alpha+\beta) =$ ____________________

b. Rewrite the left side of the equation in step a, replacing β with α : _______________ $= cos2a$

c. Since we replaced β in the left side, we need to do the same on the right. We get:

____

↑ Be sure to simplify (there are terms you can combine)! ↑

d. Put the results together to get our first double-angle formula for cosine:

$cos2a =$ $cos^2\alpha - sin^2\alpha$

e. Cosine is a bit different, as it has multiple forms of the double-angle formula. To find a different form, use the trigonometric identity $sin^2\alpha + cos^2\alpha = 1$.

f. Using the trigonometric identity, above, solve for $cos^2\alpha =$ _________________

g. In the double-angle formula that you discovered after part d, replace $cos^2\alpha$ with the results in part f. We get:

____.

↑ Be sure to simplify (there are terms you can combine)! ↑

h. Put the results together to get our second double-angle formula for cosine:

$$cos2a = 1-2\sin^2\alpha$$

i. There is still one more form of the double-angle identity for cosine. This time, use the trigonometric identity in part e to solve for $sin^2\alpha$ = ____________________

j. In the double-angle formula that you discovered after part d, replace $sin^2\alpha$ with the results in part i. We get:

____.

↑ Be sure to simplify (there are terms you can combine)! ↑

k. Put the results together to get our third double-angle formula for cosine:

$$cos2a = 2\cos^2\alpha - 1$$

Part 4: Tangent of a double-angle→ Tan( + )

a. Recall from the last section, the tangent of the sum of two angles: $tan(\alpha+\beta) =$ ____________________

b. Rewrite the left side of the equation in step a, replacing β with α: _______________ $= tan2a$

c. Since we replaced β in the left side, we need to do the same on the right. We get:

____

↑ Be sure to simplify (there are terms you can combine)! ↑

d. Put the results together to get our first double-angle formula:

$$tan2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

Part 5: Summary

Complete the Table ↓

$sin2\alpha =$	$cos2a =$ (Be sure to list all 3)	$tan2\alpha =$

Discovering the Power-Reducing Formulas

Yesterday, we discovered that double-angle formulas. These double angle formulas can be used to obtain the power reducing formulas! These formulas allow us to reduce the power of trig functions with even degrees.

Part 1: Power-Reducing Formula for $cos^2\theta$

Step 1: Rewriting the important equations that we will use

a. $cos(2\theta) =$
b. $cos^2\theta + sin^2\theta =$

Step 2: Add these two equations together

$$\begin{aligned} cos2\theta &= cos^2\theta - sin^2\theta \\ + \quad 1 &= cos^2\theta + sin^2\theta \end{aligned}$$

Step 3: Solve for $cos^2\theta$, and we obtain the power-reducing formula for $cos^2\theta$

$cos^2\theta$ =

Part 2: Power-Reducing Formula for $sin^2\theta$

Step 1: Same as part 1

Step 2: Subtract these two equations together

$$\begin{aligned} 1 &= cos^2\theta + sin^2\theta \\ - \quad (cos2\theta &= cos^2\theta - sin^2\theta) \end{aligned}$$

Step 3: Solve for $sin^2\theta$, and we obtain the power-reducing formula for $sin^2\theta$

$sin^2\theta$ =

Discovering the Half-Angle Formulas

Some alternative forms of the power-reducing formulas come from replacing θ with $\frac{\theta}{2}$, creating the half-angle formulas.

Part 1: Half-Angle Formula for $cos\frac{\theta}{2}$

Step 1: Rewrite the formula for $cos^2\theta$, below

Step 2: Replace each θ with $\frac{\theta}{2}$

Step 3: Solve for $cos\frac{\theta}{2}$

depends on original angle

Conclusion:

$$cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

Part 2: Repeat steps 1-3 to discover the formulas for $sin\frac{\theta}{2}$ and $tan\frac{\theta}{2}$

Conclusion:

$sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$	$tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

Example 1: Use half-angle formulas to find the exact value of $sin 105^{\circ}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\sin\frac{210}{2} = +\sqrt{\frac{1-\cos 210}{2}}$

$\sqrt{\frac{1--\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}}$

$\sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

Example 2: Find all solutions of $1 + cos^2x = 2cos^2\frac{x}{2}$ in the interval $[0, 2\pi)$

$2\left(\cos\frac{x}{2}\right)^2$

$2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$

$\cancel{2}\left(\frac{1+\cos x}{\cancel{2}}\right) = 1+\cos x$

$1+\cos^2 x = 1+\cos x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1)$

$\cos x = 0$ $\quad$ $\cos x - 1 = 0$

$\cos x = 1$

$\boxed{\frac{\pi}{2}, \frac{3\pi}{2} \quad 0}$

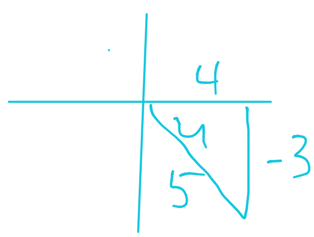
Example 3: Find the exact value of $cos2u$ using a double-angle identity given $cosu = \frac{4}{5}$ and $\frac{3\pi}{2} < u < 2\pi$

$2\cos^2 x - 1 = \cos 2x$

$2\left(\frac{4}{5}\right)^2 - 1$

$2\left(\frac{16}{25}\right) - 1$

$\frac{32}{25} - \frac{25}{25} = \frac{7}{25}$



Example 4: Find all solutions of $sin2x + cosx = 0$ in the interval $[0, 2\pi)$

$2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\quad$ $2\sin x + 1 = 0$

$\cos x = 0 \rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = -\frac{1}{2} \rightarrow \frac{11\pi}{6}, \frac{7\pi}{6}$

Example 5: Find all solutions of $sin2xsinx = cosx$ in the interval $[0, 2\pi)$

$2\sin x\cos x\sin x = \cos x$

$2\sin x\cos x\sin x - \cos x = 0$

$2\sin^2 x\cos x - \cos x = 0$

$\cos x(2\sin^2 x - 1) = 0$

$\cos x = 0$ $\quad$ $2\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$