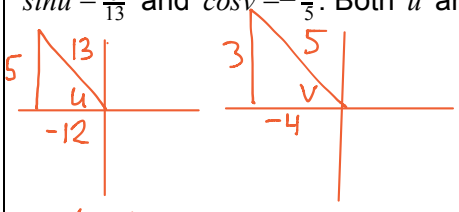
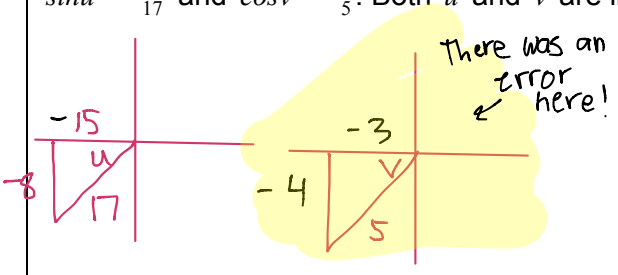
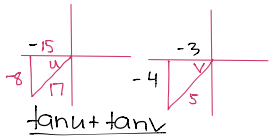
5.4 Practice

Example	Practice
1a. Find the exact value of $cos(u+v)$ given that $sinu=\frac{5}{13}$ and $cosv=-\frac{4}{5}$. Both u and v are in Q2.  $\cos(u+v)=\cos u\cos v-\sin u\sin v$ $=\left(\frac{-12}{13}\right)\left(\frac{-4}{5}\right)-\left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$ $=\frac{48}{65}-\frac{15}{65}=\boxed{\frac{33}{65}}$	1b. Find the exact value of $cos(u-v)$ given that $sinu=\frac{5}{13}$ and $cosv=-\frac{4}{5}$. Both u and v are in Q2. **BE SURE TO USE THE CORRECT FORMULA!** $\frac{63}{65}$
2a. Find the exact value of $tan(u-v)$ give that $sinu=-\frac{8}{17}$ and $cosv=-\frac{3}{5}$. Both u and v are in Q3.  $\tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$ $\frac{-8}{-15}-\frac{-4}{-3}=\frac{\frac{8}{15}-\frac{20}{15}}{1+\frac{32}{45}}=\frac{-\frac{12}{15}}{\frac{77}{45}}=\frac{-36}{45}\cdot\frac{45}{77}=\boxed{\frac{-36}{77}}$	2b. Find the exact value of $tan(u+v)$ give that $sinu=-\frac{8}{17}$ and $cosv=-\frac{3}{5}$. Both u and v are in Q3.  $\frac{\tan u+\tan v}{1-\tan u\tan v}$ $\frac{\left(\frac{8}{15}\right)+\left(\frac{4}{3}\right)}{1-\left(\frac{8}{15}\right)\left(\frac{4}{3}\right)}=\frac{84}{13}$
3a. Find the exact value of the expression: $cos(\frac{\pi}{4}+\frac{\pi}{3})$ $u=\frac{\pi}{4}\quad v=\frac{\pi}{3}$ $\cos(u+v)=\cos u\cos v-\sin u\sin v$ $\cos\frac{\pi}{4}\cos\frac{\pi}{3}-\sin\frac{\pi}{4}\sin\frac{\pi}{3}$ $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}=\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$	3b. Find the exact value of the expression: $sin(\frac{7\pi}{6}-\frac{\pi}{3})$ $u\quad v$ $\sin\left(\frac{7\pi}{6}-\frac{\pi}{3}\right)$ $\sin u\cos v-\cos u\sin v$ $\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $\frac{-1}{4}+\frac{3}{4}=\frac{1}{2}$

4a. Find the exact value of $\cos(\frac{11\pi}{12})$ given that $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$

$u = \frac{3\pi}{4}$ $v = \frac{\pi}{6}$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

4b. Find the exact value of $\sin(\frac{17\pi}{12})$ given that $\frac{17\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}$

5a. Find the exact value of $\sin(\frac{13\pi}{12})$

Now we have to decide what 2 values on the unit circle add up to $\frac{13\pi}{12}$

$u = \frac{3\pi}{4}$

$v = \frac{\pi}{3}$

$\frac{3\pi}{4} + \frac{\pi}{3} = \frac{9\pi}{12} + \frac{4\pi}{12} = \frac{13\pi}{12}$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

5b. Find the exact value of $\cos(\frac{5\pi}{12})$

$\frac{\pi}{6} + \frac{\pi}{4}$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

6a. Find the exact value of $\tan(-\frac{7\pi}{6})$

We need to choose values that add or subtract to $\frac{-7\pi}{6}$

maybe... $\frac{\pi}{3} - \frac{3\pi}{2} = \frac{2\pi}{6} - \frac{9\pi}{6} = \frac{-7\pi}{6}$ woo!

$u = \frac{\pi}{3}$, $v = \frac{3\pi}{2}$

We subtracted, so use formula for $\tan(u-v)$

$\frac{\tan u - \tan v}{1 + \tan u \tan v} = \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{3\pi}{2}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{3\pi}{2}\right)}$

$= \sqrt{3}$ - undefined

...we need to find new u and v values...

$\frac{\pi}{6} - \frac{8\pi}{6} = \frac{-7\pi}{6}$

$\frac{8\pi}{6} = \frac{4\pi}{3}$

$u = \frac{\pi}{6}$, $v = \frac{4\pi}{3}$

$\frac{\tan\left(\frac{\pi}{6}\right) - \tan\left(\frac{4\pi}{3}\right)}{1 + \tan\left(\frac{\pi}{6}\right)\tan\left(\frac{4\pi}{3}\right)}$

$= \frac{\frac{\sqrt{3}}{3} - \sqrt{3}}{1 + \left(\frac{\sqrt{3}}{3}\right)\left(\sqrt{3}\right)} = \frac{\frac{\sqrt{3}}{3} - \frac{3\sqrt{3}}{3}}{1+1}$

$\frac{\frac{-2\sqrt{3}}{3}}{\frac{2}{1}} = \frac{-2\sqrt{3}}{3} \cdot \frac{1}{2} = \boxed{\frac{-\sqrt{3}}{3}}$

6b. Find the exact value of $\sin(-\frac{13\pi}{12})$

$\frac{\pi}{4} - \frac{4\pi}{3}$

$\sin u \cos v - \cos u \sin v$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right)$

$\frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}+\sqrt{6}}{4}$