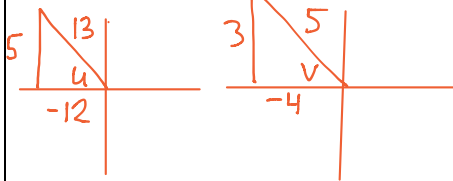


5.4 Practice

Example

1a. Find the exact value of $\cos(u+v)$ given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{4}{5}$. Both u and v are in Q2.

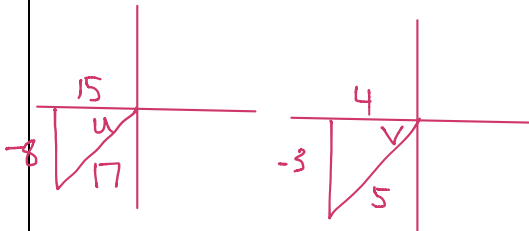


$$\begin{aligned}\cos(u+v) &= \cos u \cos v - \sin u \sin v \\ &= \left(\frac{-12}{13}\right)\left(\frac{-4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right) \\ &= \frac{48}{65} - \frac{15}{65} = \boxed{\frac{33}{65}}\end{aligned}$$

Practice

1b. Find the exact value of $\cos(u-v)$ given that $\sin u = \frac{5}{13}$ and $\cos v = -\frac{4}{5}$. Both u and v are in Q2. **BE SURE TO USE THE CORRECT FORMULA!**

2a. Find the exact value of $\tan(u-v)$ give that $\sin u = -\frac{8}{17}$ and $\cos v = -\frac{3}{5}$. Both u and v are in Q3.



$$\begin{aligned}\tan(u-v) &= \frac{\tan u - \tan v}{1 + \tan u \tan v} \\ \frac{\left(\frac{-8}{15}\right) - \left(\frac{-3}{4}\right)}{1 + \left(\frac{-8}{15}\right)\left(\frac{-3}{4}\right)} &= \frac{\frac{-32}{60} - \frac{-45}{60}}{\frac{60}{60} + \frac{24}{60}} = \frac{\frac{13}{60}}{\frac{84}{60}} = \frac{13}{60} \cdot \frac{60}{84} = \boxed{\frac{13}{84}}\end{aligned}$$

2b. Find the exact value of $\tan(u+v)$ give that $\sin u = -\frac{8}{17}$ and $\cos v = -\frac{3}{5}$. Both u and v are in Q3.

3a. Find the exact value of the expression:
 $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$$u = \frac{\pi}{4} \quad v = \frac{\pi}{3}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3}$$

$$\begin{aligned}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\ \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}\end{aligned}$$

3b. Find the exact value of the expression:
 $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$

4a. Find the exact value of $\cos\left(\frac{11\pi}{12}\right)$ given that

$$\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$$

$$u = \frac{3\pi}{4} \quad v = \frac{\pi}{6}$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos\left(\frac{3\pi}{4}\right) \cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{3\pi}{4}\right) \sin\left(\frac{\pi}{6}\right)$$

$$\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

4b. Find the exact value of $\sin\left(\frac{17\pi}{12}\right)$ given that

$$\frac{17\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$$

5a. Find the exact value of $\sin\left(\frac{13\pi}{12}\right)$ $u = \frac{3\pi}{4}$

Now we have to decide what 2 values on the unit circle add up to $\frac{13\pi}{12}$

$$v = \frac{\pi}{3}$$

$$\frac{3\pi}{4} + \frac{\pi}{3} = \frac{9\pi}{12} + \frac{4\pi}{12} = \frac{13\pi}{12}$$

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin\left(\frac{3\pi}{4}\right) \cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{3\pi}{4}\right) \sin\left(\frac{\pi}{3}\right)$$

$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

5b. Find the exact value of $\cos\left(\frac{5\pi}{12}\right)$

6a. Find the exact value of $\tan\left(-\frac{7\pi}{6}\right)$

We need to choose values that add or subtract to

$$-\frac{7\pi}{6}$$

$$\text{maybe } \frac{\pi}{3} - \frac{3\pi}{2} = \frac{2\pi}{6} - \frac{9\pi}{6} = -\frac{7\pi}{6}$$

$$u = \frac{\pi}{3}, v = \frac{3\pi}{2}$$

We subtracted, so use formula for $\tan(u-v)$

$$\frac{\tan u - \tan v}{1 + \tan u \tan v} = \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{3\pi}{2}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{3\pi}{2}\right)}$$

$$= \sqrt{3} - \text{undefined}$$

... we need to find new u and v values...

$$\frac{\pi}{6} - \frac{8\pi}{6} = -\frac{7\pi}{6}$$

$$\frac{8\pi}{6} = \frac{4\pi}{3}$$

$$u = \frac{\pi}{6}, v = \frac{4\pi}{3}$$

$$\tan\left(\frac{\pi}{6}\right) - \tan\left(\frac{4\pi}{3}\right)$$

$$\frac{1 + \tan\left(\frac{\pi}{6}\right)\tan\left(\frac{4\pi}{3}\right)}{1 + \tan\left(\frac{\pi}{6}\right)\tan\left(\frac{4\pi}{3}\right)}$$

$$= \frac{\frac{\sqrt{3}}{3} - \sqrt{3}}{3} = \frac{\sqrt{3} - 3\sqrt{3}}{3}$$

$$1 + \left(\frac{\sqrt{3}}{3}\right)(\sqrt{3}) \quad 1 + 1$$

$$\frac{-2\sqrt{3}}{3} = \frac{-2\sqrt{3}}{3} \cdot \frac{1}{2} = \boxed{\frac{-\sqrt{3}}{3}}$$

6b. Find the exact value of $\sin\left(-\frac{13\pi}{12}\right)$