

5.1 Using Fundamental Identities

As you complete the table below, rate each of the groups of identities with a 3, 2, or 1.

3- I'm very familiar with this.

2- I'm somewhat familiar with this.

1- I've never seen this before...ever.

Reciprocal Identities: $\sin u = \frac{1}{\csc u}$ $\tan u = \frac{1}{\cot u}$ $\sec u = \frac{1}{\cos u}$ $\cos u = \frac{1}{\sec u}$ $\csc u = \frac{1}{\sin u}$ $\cot u = \frac{1}{\tan u}$		
Quotient Identities: $\tan u = \frac{\sin u}{\cos u}$ $\cot u = \frac{\cos u}{\sin u}$		
Pythagorean Identities: $\sin^2 u + \cos^2 u = 1$ $1 + \tan^2 u = \sec^2 u$ $1 + \cot^2 u = \csc^2 u$ $\sin^2 u = 1 - \cos^2 u$ $\cos^2 u = 1 - \sin^2 u$		
Cofunction Identities: $\sin(\frac{\pi}{2} - u) = \cos u$ $\cos(\frac{\pi}{2} - u) = \sin u$ $\tan(\frac{\pi}{2} - u) = \cot u$ $\csc(\frac{\pi}{2} - u) = \sec u$ $\sec(\frac{\pi}{2} - u) = \csc u$ $\cot(\frac{\pi}{2} - u) = \tan u$		
Even/Odd Identities **Provide an example for each $\sin(-u) = -\sin u$ odd $\csc(-u) = -\csc u$ $\cos(-u) = \cos u$ even $\sec(-u) = \sec u$ $\tan(-u) = -\tan u$ odd $\cot(-u) = -\cot u$ $u = \frac{\pi}{2}$ $\sin(-\frac{\pi}{2}) = -1$ $-\sin \frac{\pi}{2} = -1$		

Simplifying Trigonometric Identities

- Simplify $\sin x \cos^2 x - \sin x$

$$\sin x (\cos^2 x - 1)$$

$$\sin x (-\sin^2 x) = \boxed{-\sin^3 x}$$

- Simplify $\sin x + \cot x \cos x$

$$\sin x + \frac{\cos x}{\sin x} \cdot \cos x$$

$$\sin x + \frac{\cos^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \boxed{\csc x}$$

- Simplify $\cos x \tan x$

$$\cos x \frac{\sin x}{\cos x} = \boxed{\sin x}$$

Factoring a Trigonometric Expression

- Factor $\csc^2 x - \cot x - 3$

$$1 + \cot^2 x - \cot x - 3$$

$$\cot^2 x - \cot x - 2 \quad x^2 - x - 2$$

$$\boxed{(\cot x - 2)(\cot x + 1)} \quad (x-2)(x+1)$$

- Factor $\sec^4 x - \tan^4 x$

$$\frac{1}{\cos^4 x} - \frac{\sin^4 x}{\cos^4 x} = \frac{1 - \sin^4 x}{\cos^4 x} = \frac{\cos^4 x}{\cos^4 x} = \boxed{1}$$

OR $1 + \tan^4 x - \tan^4 x = \boxed{1}$

- Factor $\cot^2 x - \cot^2 x \cos^2 x$

$$\cot^2 x (1 - \cos^2 x)$$

$$\cot^2 x (\sin^2 x)$$

$$\frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x = \boxed{\cos^2 x}$$

Rewriting a Trigonometric Expression

- Rewrite $\frac{1}{1 + \sin x}$ so that it is not in fractional form.

$$\frac{1}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x} = \frac{1 - \sin x}{1 - \sin^2 x} = \frac{1 - \sin x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} = \sec^2 x - \tan x \sec x$$

OR $\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos^2 x}$

- Rewrite $\frac{\sin^2 y}{1 - \cos y}$ so that it is not in fractional form.

$$\frac{1 - \cos^2 y}{1 - \cos y} = \frac{(1 - \cos y)(1 + \cos y)}{1 - \cos y} = 1 + \cos y$$

OR $\frac{\sin^2 y}{1 - \cos y} = \frac{1 - \cos^2 y}{1 - \cos y} = \frac{(1 - \cos y)(1 + \cos y)}{1 - \cos y} = 1 + \cos y$

- Rewrite $\frac{\sin x}{\tan x}$ so that it is not in fractional form.

$$\frac{\sin x \cdot \cos x}{\sin x} = \boxed{\cos x}$$

Using Identities to Evaluate a Function

- Use the values $\sec \theta = -\frac{3}{2}$ and $\tan \theta > 0$ to find the values of all six trigonometric functions.

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{-\frac{2}{3}} = -\frac{3}{2}$$

- Use the values of $\sec \theta = \sqrt{2}$ and $\sin \theta = \frac{1}{\sqrt{2}}$ to find the values of all six trigonometric functions.

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\tan \theta = -1$$

$$\cot \theta = -1$$

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