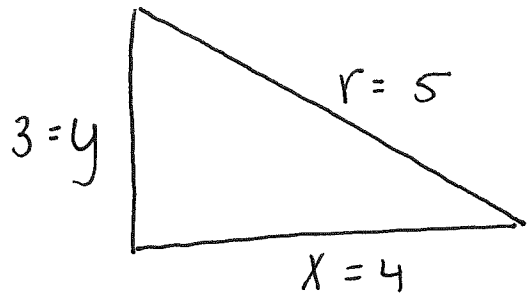
Related Rates #2B Solution



a. $\sqrt{3^2+4^2} = \boxed{5 \text{ km}}$

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b. We want to find $\frac{dr}{dt}$

Formula that relates x, y, and r → $x^2+y^2=r^2$

Given: $x = 4$ $\quad \frac{dx}{dt} = -15$ → (it's negative because traveling west)

$y = 3$ $\quad \frac{dy}{dt} = 10$ → (it's positive because traveling north. Remember, rates measure velocity, and we are given speed. $\frac{dx}{dt}$ and $\frac{dy}{dt}$ is velocity, which can be + or − depending on direction of travel).

$r = 5$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$

$$2(4)(-15) + 2(3)(10) = 2(5)\frac{dr}{dt}$$

$$-120 + 60 = 10\frac{dr}{dt}$$

$$-60 = 10\frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = -6}$$

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c. We want to find $\frac{d\theta}{dt}$

We know $\tan\theta = \frac{y}{x}$, so we will take derivative of that.

$\sec\theta = \frac{5}{4}$

$$\sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

$$\frac{25}{16}\frac{d\theta}{dt} = \frac{4(10) - (3)(-15)}{16}$$

$$\frac{25}{16}\frac{d\theta}{dt} = \frac{85}{16}$$

$$\frac{d\theta}{dt} = \frac{85}{\cancel{16}} \cdot \frac{\cancel{16}}{25} = \boxed{\frac{17}{5} \text{ radians/hr}}$$